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THE MATHEMATICS TEACHER

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Mathematics of the Aircraft Industry

By JOHN VIALL

Douglas Aircraft Company, Incorporated, Long Beach, California

THE TITLE of this discussion is a very broad subject and it is impossible to cover it completely. It would require a multi-volumed series to discuss in detail all of the applications of mathematics used in the aircraft industry; I certainly cannot do complete justice to the topic in a reasonable length of time. For this reason, I shall confine my remarks largely to general applications of specific mathematical methods of computing in the engineering of aircraft. Further, I will emphasize the type of work which is applicable to automatic computing equipment.

For those who are unfamiliar with such equipment I wish to distinguish between what is referred to as automatic equipment and desk calculators. With hand calculators, there must be an operator in constant attendance with the machine, inserting information, punching buttons to perform certain functions, and recording results. Automatic equipment has the adadvantage of having a built in operator, i.e., the machine itself is capable of inserting information, computing functions, and recording results automatically once it is started.

The most basic and widely used equipment of this type today is punched card equipment such as that manufactured by the International Business Machines Corporation. There are other bigger and more powerful machines today which have been glamorized and popularized by magazine articles. It should be pointed out at this point that the equipment referred to here is digital equipment, in other words, it operates with numbers.

I might mention another type of machine which has been extremely useful to the aeronautical engineer and that is the analog computer. This type of machine is not digital but is set up so that it represents a physical system in terms of voltages or a mechanical device, and in this way simulates the actual systems.

The computing machines referred to are mentioned because they are of extreme importance as an aid to modern science. Computers enable the engineers, scientists and mathematicians to study many things which they would otherwise not have time. Many business concerns are spending large sums of money for computing equipment.

In order to gain some idea of where and how mathematical problems arise in the engineering of an airplane let us follow the genetic stages of the development of an airplane from conception to actual production.

The customer makes known the general type of aircraft which is desired by setting down certain specifications for such items as range, gross weight, pay load, and desired speed. With this information available the preliminary design people start work designing, in broad outline, an aircraft which will meet these specifica-

tions. In many cases several configurations are actually proposed. At this point most computing is done by slide rule or "guess-timating."

If a contract is awarded, the entire engineering department then starts designing this aircraft in order to make it a flying reality. The design of a commercial plane or heavy transport, such as the DC-6 or C-124, takes from two to five years and millions of engineering man hours before the completion of the prototype. It is imperative that close coordination between the different groups be maintained at all times, so that each group can engineer their part of the plane to fit into the integrated whole.

The aerodynamics group determines the particular configuration or shape to be used. They are concerned with establishing the performance and general flying characteristics of the airplane so that they meet with the required specifications.

The aerodynamacist considers the airplane to be a rigid body moving through a fluid in a steady state condition. The performance equations he derives from Newton's law, namely that, ($F=ma$) force is equal to mass time acceleration. This of course leads to a differential equation. This equation may be of any order since the acceleration is a function of velocity. There are many of these equations in studying the performance of an airplane. Being familiar with mathematics you know that differential equations for which an analytic solution can readily be found are rather restricted in number. As is the case with most physical problems, these performance equations are no exception and therefore an analytic solution is not readily obtained.

It is at this point that the engineer must call upon his ingenuity and experience to help the mathematician on the problem. From his experiences he can make assumptions which will be true for the particular problem being studied. The mathematician, in certain instances, may formulate approximating functions which will permit an analytical solution of the differential equations. Both assumptions and approximations have been used in deriving analytical expressions in the performance equations.

The results obtained from these performance calculations will actually determine such items as the air foil, fuselage, wing plan, and tail assembly in the final configuration. An enormous number of calculations must be made in order to verify that the performance of the airplane will meet specifications.

A great amount of work has been done on performance and there is even more to do. The greatest problem which was presented was due to the fact that the aerodynamist had previously obtained much information from graphs. Therefore it was necessary to fit approximating functions to this graphical data. Most of these approximations have successfully been accomplished with polynomials or rational fractions. Of particular interest is the equation of the form

$$y=\frac{kx^a}{y^b z^e w^e}$$

where it was required to find k and the exponents a, b, r, and e. In this particular case, the original data was flight test data and hence in fitting the curve the method of least squares was used as it was desirable to smooth the data.

Although the aerodynamacist considers the airplane to be a rigid body moving through a fluid in a steady state condition, it has long been realized that this is not the actual condition. The airplane is an elastic body and rather than a steady state response, a transient response is the actual condition. During the recent past, powerful methods of analysis have been developed which make possible the study of the dynamic properties of the airplane. For this reason, most major aircraft companies have incorporated within the engineering department a dynamics group. The purpose of this group is to study problems such as flutter, vibration, and

dynamic stability and control. Due to the lack of cut and dried methods of solution to these problems, much time is spent by this group in developing methods of analysis which will best describe the transient behavior of the airplane. Also the advent of high speed aircraft have made these problems one of the most critical facing the designer.

The dynamics group, for most problems considers the airplane as a servomechanism with feed-back circuits. The computations for most of these analyses, as would be expected, involves the solution of simultaneous differential equations, describing the response of the aircraft to a given input. These differential equations are solved by the use of automatic computers using, in most cases the methods set forth by Milne in his book *Numerical Calculus.*[1] The Kutta-Runge[2] method is also quite useful due to the fact that values other than initial conditions are unnecessary to start the solution. A method developed by Madwed[3] in writing his doctorial thesis at Massachusetts Institute of Technology, "The Number Series Method of Solution of Linear and Non-Linear Differential Equations" has been used to great advantage in the numerical solution of differential equations.

One of the greatest needs today is to find simple numerical methods of solution to non-linear differential equations. The fact that non-linear differential equations methods often are messy, if not impossible to obtain, has limited the dynamacist to assumptions of linear systems only, when the actual system could best be described by a system of non-linear equations.

Matrix algebra is used extensively in the investigation and analysis of flutter and vibration phenomena. Matrices permit a large amount of information to be recorded in a short time and furthermore provide a ready made coding system. Of perhaps greater advantage, matrix manipulations are easily adapted to automatic computing methods, and with little extra effort a self checking operation can be incorporated into the matrices. This checking method is the check sum method which is used very extensively in computing. It consists of adding to the multiplier matrix a row, called the row of check sums, which are the negative sums of the corresponding columns. To the multiplicand matrix is added a column, called the column of check sums, which are the negative sums of the corresponding rows. Upon multiplication of these matrices we obtain a produce matrix with an extra row and column, which are called the check sums. If the multiplication has been performed correctly, the product matrix rows and columns will sum to zero. There may be, however, small errors which are due to the rounding of the individual products. Mathematic checks, utilizing this same idea, are used when performing other matrix computation.

The modes of flutter and vibration in the airplane as a whole or a specific part are represented in matrix form. In order to determine the individual modes of flutter, or of vibration, it is necessary to solve a matrix for its characteristic roots and vectors. The order of the matrix is directly dependent upon the number of modes present which the analyst considers critical. The matrix representing the modes of vibration is usually a symmetric matrix, whereas the matrix representing the flutter modes is non-symmetric and complex. The best method for solving symmetric matrices, with respect to machine computation at least, is the gradient method developed by Drs. Hestenes and Karush[4] of the Institute for Numerical

[1] W. E. Milne, *Numerical Calculus.* Princeton, N. J.: Princeton University Press, 1949.

[2] C. Runge and H. König, *Vorlesungen Über Numeriches Rechnung,* ("Die Grundlehren der Mathematischen Wissenschaften," Vol. XI, [Berlin: Julius Springer, 1924]), Chapter X.

[3] A. Madwed, "Number Series Method of Solving Linear and Non Linear Differential Equations." (Report No. 6445-26, Instrumentation Laboratory), Cambridge: Massachusetts Institute of Technology, April 1950.

[4] M. R. Hestenes and W. Karush, "A Method of Gradients for the Calculation of the

Analysis, at the University of California at Los Angeles. This method utilizes the fact that the eigen vectors of symmetric matrices are mutually orthogonal. The gradient method has been applied to matrices of order up to the eleventh with excellent results, i.e., 7 to 8 correct significant digits in all roots and vectors where we started with 8 or 9 digits in the original matrix.

Finding the characteristic roots and vectors of non-symmetric matrices presents a much more difficult problem. In fact a great deal of investigation is needed in this field. At the present time, the method of Leverrier,[5] which consists of raising the matrix to all powers up to and including the order of the matrix, is being used. The sums of the diagonal terms of the various powers are used to form a set of simultaneous equations whose solutions are the coefficients of the characteristic equation. The last part is somewhat simplified since the matrix of coefficients of the simultaneous equations is a triangular matrix. The characteristic equation must then be solved for the eigen values. To obtain the eigen vectors it is necessary to set up N sets, where N is the order of the original matrix, of Nth order simultaneous equations and solve these N sets for the N eigen vectors. This is a large computational job, even with use of automatic computing equipment. If the elements of the original matrix are complex, as is the case of the flutter matrix, the work is increased many fold.

The computation necessary to form the characteristic equation may be reduced by employing a method due to Krylov.[5] This method consists of a series of post multiplications of the matrix by an arbitrary column matrix; forming a set of linear simultaneous equations, the solutions of which will yield the coefficients of the characteristic equation. It should be pointed out, that due to the excessive amount of arithmetic operations it is hard to retain accuracy throughout the entire process for either of the aforementioned methods.

Another important group in engineering is the stress group or sometimes called the strength group. Their work is consistent with their name as their primary job is to check the strength of the structural components of the airplane. The work consists of writing the preliminary and formal stress and load analysis reports needed to satisfy the customer as to the strength of the airplane.

In determining the normal and shear stress which each structural member of the airplane must carry, the plane is considered as a rigid body, or beam, and all loading conditions are investigated. Before the advent of computing equipment, the stress analyst was limited, because of the prohibitive amount of computation to the investigation of two or three loading conditions which he believed to be the most critical. With high speed automatic computers, the stress analyst can now consider all loading conditions which might be critical and with this information he is able to determine more accurately what the strength of a particular structural member must be. In the setting up of loading conditions air loads, gust loads, inertia loads and accelerations loads are considered separately and in combinations. As many as 150 different loading conditions may be considered as critical or near-critical in computing the necessary strength values which the structural members of the airplane must possess.

Weight is one of the prime factors in the building of airplanes. Because of this it is the responsibility of the stress group to recommend materials that are strong enough for the loads that must be carried. It is just as important however that they are not over strength for this increases weight and hence cuts down the pay load

Characteristic Roots and Vectors of a Real Symmetrix Matrix," *National Bureau of Standards Journal of Research*, XLVII (July 1951), 45–61.

[5] H. Wayland, "Expansion of Determinantal Equations into Polynomial Form," *Quarterly of Applied Mathematics*, II (January 1945), 277–306.

which may be carried. More serious is the fact that overstrengthening of a member may transmit a load to another member which was not designed for this load.

Most of the computation in the evaluation of strength values consists of the evaluating of simple algebraic formulae or finding the solution of simple differential equations. One exception to this is the computing of the margins of safety which must be reported to the customer. This involves solving the equation $kx^{7/4}+mx-1=0$ where k is a parameter. This was done by an iterative method developed by a member of our staff. It was known that the solution in which we were interested was in the range zero to one. It was determined that the family of curves looked something like Figure 1 in which the y-intercept was always the point $(0, -1)$. An arbitrary point below the y-intercept was chosen and the equation of the straight line through it and the point $[1, f(1)]$ was solved for its x intercept, (x_1). The equation of the line through $[1, f(1)]$ and $[x_1, f(x_1)]$ was computed and again solved for the x intercept, (x_2). This was repeated until the $f(x_i)$ was zero, which would then be the solution of the equation.

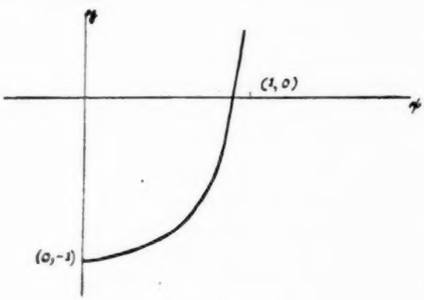

FIG. 1

As mentioned before, I am emphasizing the mathematics which the computing group has been doing. The bulk of our work has been from the three groups which I have covered. However, there have been some other work which we has done for other engineering groups.

The equipment group is concerned with all non-functional equipment which is external to the air frame such as radios, antennas and other items not a part of flying functions. Among their many tasks they are responsible for the design of the radome, which is the shell that fits over the radar antenna. It is necessary to analyze the transmission characteristics of each proposed radome to establish how efficiently it transmits the electro-magnetic energy emitted by the antenna, also, how much distortion of the electro-magnetic waves take place. This problem in some respects, is similar to that in the study of optics.

To mathematically perform this analysis it requires the solution of the Lossy equation. This is done by taking ten two by two matrices with complex elements and multiplying them together. We have been of further help in that we have been able to compute the elements of the matrices themselves, which are expressed in terms of exponential, trigonometric and irrational functions. To do this problem with a slide rule or desk calculator would require one man several days to complete. This problem on automatic computing equipment takes a matter of a few minutes. To design and test one radome requires that literally hundreds of these problems be solved. From this example you can realize the number of man hours which can be saved by using computing equipment.

The primary function of the power plant group is to work out the installation problems of the power plant and related systems such as fuel systems and controls, and pneumatic system. They also test the power plant selected by the aerodynamics and preliminary design groups. Very few of the computations performed by this group are adaptable to digital computers but rather are primarily the type best solved on analog computers.

One problem which was solved on digital machines for this group concerned the insulation of the tailpipe of an aircraft

engine. In this particular problem it was not convenient to make a direct solution for the various variables. The variables were permitted to vary over given ranges and an algebraic formula was evaluated for the various combinations of these variables and the optimum condition was chosen. In the solution of this problem it was necessary to solve a fourth degree polynomial. Some four thousand (4,000) of these equations were solved in approximately two (2) hours, using Newton's method.

In the design of an aircraft, a theoretical chord called the "Mean Aerodynamic Chord" is determined. The c.g., center of gravity, of the airplane must fall in the narrow limits of this chord in order that the aircraft may have good flying characteristics. For this reason the distribution of cargo loads is very important. The weights group is the unit which determines the optimum storage of cargo. In determining this, it is necessary that they know where the c.g. is located with every conceivable type of loading condition in the airplane.

In order to determine accurately the c.g. it is imperative that they know the weight of each part of the aircraft. These weights go through a series of steps from estimated to calculated and finally to actual weights. In the early stages of design, accurate estimations and calculations of weight are a great help to the designer. This of course involves the calculation of weights for regular and irregular shaped bodies. In computing the c.g. of the airplane thousands upon thousands of calculations are made in determining and summing the moments of the individual parts.

An interesting problem of center of gravity was recently handled on automatic computing equipment. Involved was an irregular shaped fuel tank for which was desired the center of gravity for any level of fuel in the tank.

In the representative figure (Fig. 2), the end of the tank represented by the curve AB is in reality hemispherical in shape. We were given seven points on this curve through which it was necessary to fair an approximating function. In order to have a desired accuracy in the results, it was necessary to fit the curve AB with two quartic polynomials. The center of gravity was found by computing areas and moments of small layers, summing them, and then with simple division steps, computing the c.g. Symmetry of the tank about a reference plane made this easier since it was necessary only to work with areas in a plane.

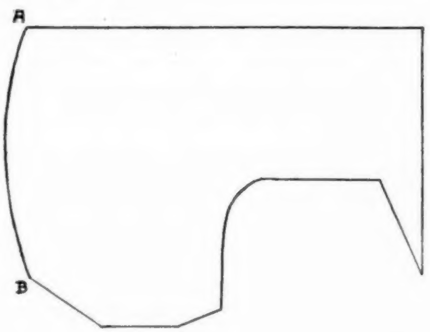

FIG. 2

The lofting group although not a part of engineering has a large number of calculations to perform. This group sets up the loft lines which determines the contour of the airplane and from which the tooling departments are able to design the tools necessary to produce the aircraft. This work is done in the early stages of the design and the engineering department is very dependent upon it.

A large part of their work concerns the fitting of curves, usually conics, through a set of given points. Once this curve is determined it is necessary to compute points along the curve at one inch increments. In doing this, automatic computing equipment has been of great service to this group.

There are other groups in the engineering department whose functions in the design of an airplane are very important. As of yet, little of their computation has

been adapted to digital computing machines and much of this is more adaptable to analog computers.

The solution of simultaneous equations has been mentioned in several places. The method used extensively in computing is the Jordan Method. This method is a version of the well known reduction method, whereby the matrix of coefficients is reduced by elementary row transformations to a unit matrix. This same method is applied to the finding of the inverse of a matrix, where the matrix of row transformations necessary to reduce the original matrix to the identity matrix represents the inverse of that matrix.

The methods of solution of polynomial equations varies with the particular problem. For all polynomials of degree higher than the second, an iterative scheme of solution is generally more adaptable to machine computation. Of all the methods available, Newton's method is perhaps the most popular. This consists of obtaining a new approximation to the root by subtracting from the previous approximation, the quotient of the function evaluated at the previous approximation and the value of the first derivative of the function evaluated at the previous approximation. The sentence above is a choice bit isn't it; it is a prime example of how it is possible to replace a truck load of words with a few mathematical symbols. The above sentence could, and should, be said thusly:

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}\,.$$

In most cases this scheme converges to one of the roots of the polynomial. Bairstowe's[6] method and Linn's[7] method also find widespread usage. These two methods rely upon simple formula to correct the first approximation to get the second approximation, etc. Whereas Newton's method solves for a single root, these two methods solve for a quadratic factor of the polynomial. All of these methods have limitations and their usefulness must be determined by the analyst in setting up the problem. The problem of root finding is greatly intensified when the coefficients of the polynomial lie in the complex plane. At present, Newton's method is perhaps the best, but it would seem possible that an easier method could be developed.

At the present time there is a very great need for additional work and investigation in the field of Numerical Analysis. There is also a need for trained personnel in this same field. It is one which has received little recognition until recently. Many universities throughout the country are now offering courses which emphasize Numerical Analysis. In fact, this year, for the first time, the University of California is offering in its Extension Department a certificate in Numerical Analysis. It is a relatively new field which is growing and offers individuals a chance to do original work early in their careers as well as being well rewarded. I believe you can perform a great service by guiding those pupils of yours who are interested in mathematics into the Numerical Analysis field.

[6] T. C. Fry, "Some Numerical Methods for Locating Roots of Polynomials," *Quarterly of Applied Mathematics*, III (July 1945), 89–105.

[7] Proceedings of Computation Seminar, International Business Machine Corporation, All Editions.

Notice of Annual Business Meeting

As required by the By-laws of the National Council of Teachers of Mathematics, notice is hereby given to the members that the annual business meeting will be held Saturday, April 11, 1953 at the Hotel Ambassador, Atlantic City, New Jersey.

M. H. AHRENDT, *Executive Secretary*

Can We Outdo Mascheroni?

By WM. FITCH CHENEY, JR.
University of Connecticut, Storrs, Connecticut

LORENZO MASCHERONI was for many years professor of mathematics at the University of Pavia (some twenty-two miles south of Milan), where Christopher Columbus had once been a student. Mascheroni was an Italian. He was born in 1750 and died in 1800. During his life, he published a considerable number of mathematical writings, the best known of which was his *Geometry of the Compass*, which first appeared in 1797. In it he showed how all standard constructions usually performed with straightedge and compass could be carried out with the compass alone. Mascheroni claimed that the compass was more accurate than the straightedge, since few, if any, "straight-edges" are really straight, and they tend to skid more easily than a compass when in use.

Mascheroni used the "Modern Compass," which retained its setting when lifted from the paper, rather than the more elegant "Classical Euclidean Compass of the Greeks," which would close up if either point was raised from the drawing surface. In his constructions, Mascheroni frequently reflected points across lines, but did not know of inversion, which was discovered by Steiner in 1824, and was later named by Liouville "the Transformation by Reciprocal Radii."

As is well known, the mechanics of reflecting the point, P, across the line, AB, consists in swinging two classical arcs through P, centered at A and B respectively, until they meet again at P'. Inversion, on the other hand, replaces P by Q on the same produced radius of the inversion circle centered at O, such that the radius, r, of that circle is the geometric mean of OP and OQ. If $OP=r$, so does OQ. If $\frac{1}{2}r<OP$, the mechanics of inversion consists in drawing $P\circ O$, (that is, the circle centered at P and through O), to intersect the inversion circle at S and T, and then reflecting O across ST to Q. This construction is readily justified through the similar isosceles triangles, SOP and OQS. It requires three classical arcs. If P is less than $\frac{1}{2}r$ distant from O, its distance from O must be doubled, to P_1, by letting $O\circ P$ cut $P\circ O$ at U and V, and $U\circ V$ cut $P\circ O$ at P_1. If P_1 is more than $\frac{1}{2}r$ from O, it is inverted to Q_1 by the process outlined above, and Q_1 then replaced by Q, twice as far from O. If P_1 is not more than $\frac{1}{2}r$ from O, doubling must continue to P_n, which is. After inverting P_n to Q_n, the distance of the latter from O must then be doubled the same number of times to reach Q, the inverse of the original P. To double any distance with compass only takes three classical arcs.

In 1890, Professor August Adler of Vienna published a book on *The Theory of Geometric Constructions*, and devoted Chapter Three to the consideration of Mascheroni's constructions. Professor Adler used the classical compass throughout his work, and relied largely on inversion. He could do everything with the classical compass that Mascheroni could do with his modern one.

Still later, in 1916, Hilda P. Hudson published a book on *Ruler and Compasses* (Longmans, Green and Co.), in Chapter Eight of which she commented on the work of Mascheroni and Adler, and focused attention on the number of arcs necessary in their various constructions, and whether or not they were all classical. It is largely from the stimuli of these publications that the present paper has been produced.

It is said that Napoleon Bonaparte delighted in stumping his engineers with the problem of quadrisecting a given circumference with a compass as the only tool. This problem is now generally known as

"Napoleon's Problem," although it had been known before Napoleon's time. Mascheroni solved this problem by first stepping off the radius of the given circle around its circumference to locate in succession the four points, A, B, C and D, at 60° intervals. (See Fig. 1. In the figures of this paper, all construction arcs are numbered in the order in which they are drawn.) Thus AD was a diameter. He then let $A \circ C$ cut $D \circ B$ at E and F. If the center of the given circle was called O, he then swung an arc centered at A and with radius equal to OE, to cut $O \circ A$ at G and H. This resulted in the arcs, $AH = HD = DG = GA = 90°$. To justify this construction, note that in triangle AOC the cosine law tells us that $AC = r\sqrt{3}$. The law of Pythagoras applied to right triangle AOE makes $OE = r\sqrt{2}$, and in any circle, $r\sqrt{2}$ is the length of the chord to subtend a 90° arc. This construction of Mascheroni to solve Napoleon's Problem requires six arcs, the last of which is necessarily modern.

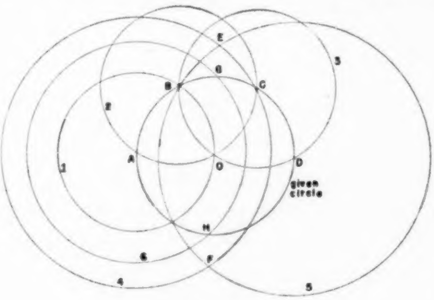

Fig. 1

If we wished to solve Napoleon's Problem with the classical compass, we could replace Mascheroni's last step by reflecting E across BB' to E' and drawing $A \circ E'$ to cut $O \circ A$ at G and H. (Here B' is the other point of intersection of $O \circ A$ with $A \circ O$. See Fig. 2.) This quadrisects the circumference with eight classical arcs. However, we note that the circle $O \circ E$ completes the quadrisection of circle $B \circ O$, with the point diametrically opposite O lying on $A \circ C$. Hence we may assume that $B \circ O$ is given, (See Fig. 3), and construct in turn $O \circ B$, $C \circ B$, $A \circ C$, $D \circ B$ and $O \circ E$ to quadrisect $B \circ O$ by the points, O, P, Q and R. This requires only five arcs, and they are all five classical. (It is of interest in passing to note that this construction simultaneously quadrisects the circumference of $C \circ O$.)

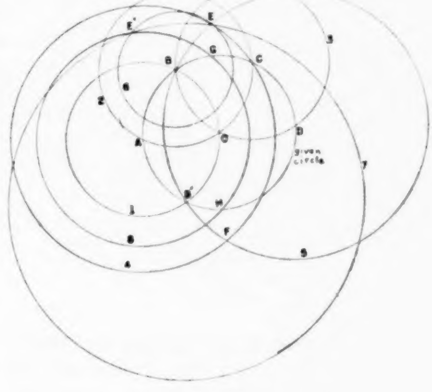

Fig. 2

A second and ancient problem solved by Mascheroni with compass only, was to construct the lengths of the sides, d and p, of the regular decagon and pentagon inscriptible in a given circle, whose radius may be taken as unit of measure. Mascheroni first quadrisected the given circle, $O \circ A$, as described above. (See Fig. 4.) He then let $G \circ O$ cut $O \circ A$ at J and K so that arcs $AJ = JB = BG = GC = CK = KD = 30°$. Finally he let $J \circ C$ cut $K \circ B$ inside $O \circ A$ at L. This construction makes $OL = d$ and $AL = p$, as is shown in the following paragraphs.

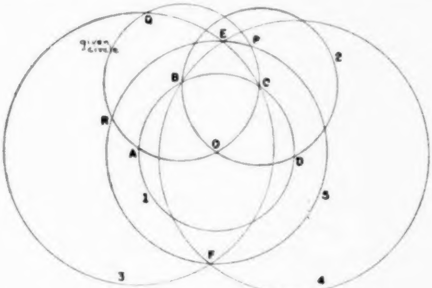

Fig. 3

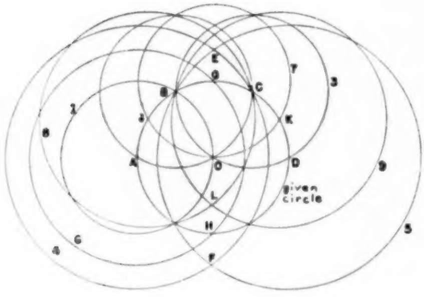

FIG. 4

If, in a circle of unit radius, we inscribe a regular decagon of side d, then one side and two radii to its ends form an isosceles triangle, ABO, whose angles are $36°$, $72°$ and $72°$. (See Fig. 5.) If AC bisects angle A, then triangles ABC and ACO are isosceles, with triangle ABC similar to triangle ABO. From this similarity arises the quadratic equation, $d^2+d=1$, whose positive root is the famous Fibonacci ratio, $(\sqrt{5}-1)/2$. Now in Figure 4, the sides of triangle JKL are by construction $\sqrt{3}$, $\sqrt{2}$ and $\sqrt{2}$. Hence its altitude from L is $\frac{1}{2}\sqrt{5}$, and $LO=\frac{1}{2}\sqrt{5}-\frac{1}{2}=d$.

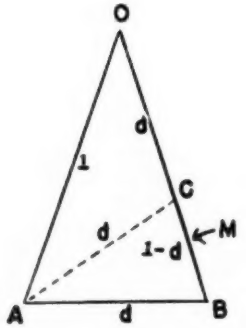

FIG. 5

Since connecting alternate vertices in a regular decagon produces a regular pentagon, we see that the perpendicular from A to BC, in Figure 5, would bisect it at M. Then $AM=\frac{1}{2}p$ and $MC=\frac{1}{2}(1-d)$. In the right triangle, AMC, $AM^2+MC^2=AC^2$, or $p^2/4+(1-d)^2/4=d^2$. Since $d=1-d^2$, this reduces to $p^2=1+d^2$, so that p is the hypothenuse of a right triangle whose legs are 1 and d. But in Figure 4, ALO is a right triangle with legs 1 and d, so its hypothenuse, $AL=p$.

In this construction, Mascheroni used a total of nine arcs, one of which was modern. Figure 6 illustrates an alternative construction to improve on this plethora. With $O \circ A$ given, let $A \circ O$ cut $O \circ A$ at B and C. Let $B \circ O$ meet $A \circ O$ again at D, and meet $O \circ A$ again at E. Let $C \circ B$

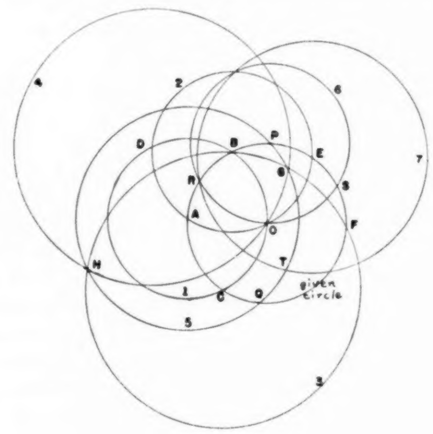

FIG. 6

meet $O \circ A$ again at F, and meet $D \circ O$ at G and H. Then $A \circ G$ meets $O \circ A$ at P and Q, such that arcs $AQ=QF=FP=PA=90°$. Let $P \circ O$ meet $O \circ A$ at R and S, and let $E \circ R$ meet $A \circ G$ inside $O \circ A$ at T. Then $OT=d$ and $ST=p$. In this construction, all the arcs used are classical, and there are only seven of them.

A third fundamental compass construction contributed by Mascheroni was the location with compass only of the midpoint of an arbitrary arc of a given circle with known center. (See Fig. 7.) When given the arc AB on $O \circ A$, he would first draw $A \circ O$ and $B \circ O$, and cut them at C and D respectively by a circle centered at O and with radius equal to chord AB, so that CD was a diameter of this last circle. He then let $C \circ B$ cut $D \circ A$ at E. Lastly he swung an arc centered at C, with radius equal to OE, to cut arc AB of $O \circ A$ at M. This required only six construction arcs, but the third and sixth of these necessitated the use of a modern compass. To

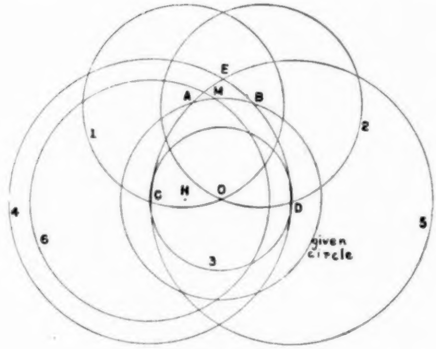

FIG. 7

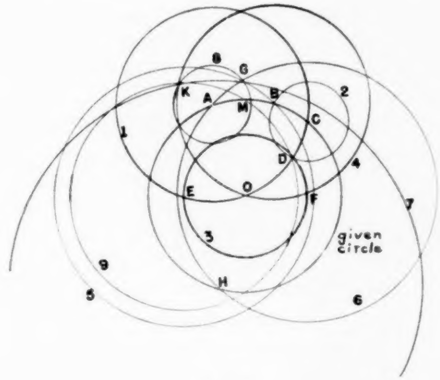

FIG. 8

justify this construction, let $OA=1$ and $AB=CO=OD=s$. Now if M is the midpoint of arc AB, COM is a right triangle of legs s and 1, so that $CM=\sqrt{1+s^2}$. Now if H is the midpoint of CO, the altitude HA of isosceles triangle COA is $\sqrt{1-s^2/4}$, and it is also the altitude of right triangle HDA, whose base is $3s/2$. Thus $AD=\sqrt{1-s^2/4+9s^2/4}=\sqrt{1+2s^2}=DE$. Finally, in right triangle ODE, $OE=\sqrt{1+2s^2-s^2}=\sqrt{1+s^2}=CM$, as desired.

Hilda Hudson solves this same problem entirely with classical arcs, for which she specifies the number necessary as fourteen. The following construction effects a definite reduction from that number. (See Fig. 8.) Assume that arc AB is given on $O\circ A$. Let $A\circ O$ cut $O\circ A$ at C. Let $C\circ B$ cut $A\circ O$ at D. Let $O\circ D$ cut $A\circ O$ at E. Let $B\circ O$ cut $O\circ D$ at F. Let $F\circ A$ cut $E\circ B$ at G, and cut $O\circ A$ at H. Reflect G across AH to K. Then $E\circ K$ bisects arc AB at M. All of the arcs used in this construction are classical, and there are only nine of them.

Mascheroni pointed out that all geometrical constructions ordinarily made with straightedge and compass were reducible to the location of a series of points found by the intersections of lines and, or, circles. Constructing the intersection of two circles is obviously independent of the use of a straightedge. The intersections of the line AB (determined by the two points, A and B) with the circle $C\circ D$, he determined by reflecting C and D across AB to E and F respectively, and drawing $E\circ F$, which must cut $C\circ D$ in the desired points. This requires only five arcs, all of which are classical. However, one arc, (and hence 20% of the work) may be saved by the following trivially obvious procedure. First reflect C across AB to E, the intersection of $A\circ C$ with $B\circ C$. Let $A\circ C$ cut $C\circ D$ at G. Let $B\circ G$ cut $A\circ C$ at H. Then $E\circ H$ will cut $C\circ D$ in the desired points. It should be noted that if A, B and C are collinear, the reflection of $C\circ D$ across AB coincides with $C\circ D$, and hence fails to determine the desired points of intersection. In this case the solution depends on bisecting the two arcs into which $A\circ D$ divides $C\circ D$, using a construction already presented in this paper.

The remaining obstacle to eliminating the necessity of the straightedge in geometrical constructions is the location of the intersection of the lines joining two given pairs of points. Mascheroni solved this problem with eleven arcs, most of which were modern. Adler solved it by inverting the two given lines with respect to an arbitrary circle, and then reinverting the proper intersection point of the two corresponding circles.* This construc-

* For the convenience of readers unfamiliar with inversion theory, the following proof is included that all the points on a straight line go into all the points on a circle thru the center of inversion. Assume that $O\circ A$ is the inversion

tion of Adler's used only classical circles, but according to Hilda Hudson, it took thirty-six of them in the general case. She reduced this number to sixteen by her construction. The present paper effects a further reduction. (See Fig. 9.)

If the two given lines are AB and CD, their point of intersection, X, may be located as follows. Reflect A across CD to E. Reflect E across AB to F. Reflect A across EF to G. Let $G \circ A$ cut $A \circ E$ at H and I. Then reflecting A across HI will give X, the desired point of intersection of AB and CD. The justification of this construction lies in the facts that a line inverts into a circle through the center of inversion, and that its center is the inverse of the reflection of the center of inversion across that line. (See footnote.) In the construction of this paragraph, the inversion circle is $A \circ E$, which coincides with $A \circ F$. The inverse of the line CD is $E \circ A$. If C and D were both reflected across AB, the resulting line, $C'D'$, would cut CD at X, and its inverse circle would, by symmetry, be $F \circ A$. But $E \circ A$ cuts $F \circ A$ at G, whose inverse must thus be X.

If AB is perpendicular to CD, the above procedure breaks down because F coincides with E. But then $A \circ E$ cuts $E \circ A$ at K and L, $K \circ L$ cuts $E \circ A$ at G, and X is found from G as before. At the most then, by the constructions just described, the intersection of two general lines may be determined by only nine arcs, all of which are classical.

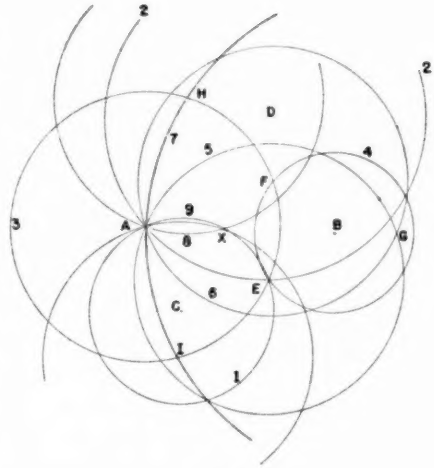

FIG. 9

The table compares the numbers of arcs used for the constructions cited. There is a challenge in the fact that further simplifications have not in all cases been proved impossible.

The problems cited above are basic. Many similar problems admit of fascinating short cuts, usually within the comprehension of the best high school students. Their investigation, where time permits, provides both a valuable review of fundamental geometrical facts and a strong stimulus to further study.

circle, that M is the foot of the perpendicular from 0 onto an arbitrary line, and that P is an arbitrary point on that line. Let N be the inverse of M, and Q of P, so that $OM \cdot ON = OQ \cdot OP$, and triangles OQN and OMP are similar. Therefore OQ is perpendicular to NQ, and as P moves on the arbitrary given line, Q moves on the circle whose diameter is ON. Furthermore, if C is the center of this circle, $OC = \frac{1}{2} ON = 1/(2 \cdot OM)$, so that C may be found by reflecting 0 across the given line and inverting the resulting point.

The problem	Number of arcs used by Mascheroni, admitting modern ones	Least previously published number of arcs, using classical arcs exclusively	Minimum number of arcs in this paper, using classical arcs exclusively
Napoleon's	6	8	5
Constructing sides of inscribed decagon and pentagon	9	11	7
Bisecting an Arc	6	14	9
Intersection of general line and circle	5	5	4
Intersection of two general straight lines	11	16	9

The Science of Teaching Mathematics

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Philosophers and professional non-teaching educators often concern themselves with the problem of involving "other values" in the teaching of mathematics. The teacher, himself, is usually more interested in the obvious and intrinsic worth of mathematics and the mathematics lesson. The purpose of this study is to review the scientific work which bears upon the teaching of mathematics in order to provide a basis for sound methodology. Further, this study aims to clear the air with respect to fads, fancies and facts in mathematics instruction; and to suggest measures for solidifying the background of pedagogical knowledge.

I. THE RESULTS OF SPECIFIC EXPERIMENTATION

The experiments to be described have been separated, arbitrarily, into seven categories for the purpose of discussion:

(1) The Organization of Instruction
(2) The Framework of Instruction
(3) Motivation of the Lesson
(4) Specific Classroom Practices
(5) The Homework Assignment
(6) Tests as Teaching Devices
(7) The Importance of the Teacher.

Wherever possible, the experiments selected have been performed with mathematics classes. In a few cases, an experiment from another field has been admitted because it was considered appropriate for our purposes. These selected experiments have, in the main, met at least the minimum requirements of valid research with respect to sampling and statistical treatment; a few, however, technically weak in one or more characteristics, have been included because of the suggestive nature of the work or because of excellence of conception. It was necessary, too, to draw new conclusions from some in which the results clearly contradicted the author's own conclusions.

(1) The Organization of Instruction

There is, at the present time, a trend toward changes in the large organization of mathematics instruction. Wren (41, p. 718) describes the results of try-outs as follows:

> **Efforts to measure the comparative values of the traditional type and the generalized type of subject matter in secondary mathematics have not been very conclusive in their results.**

Butler (6) is of the opinion, that

> . . . **efforts to teach mathematics through such plans as the activity program, socialized mathematics and the core curriculum have not been very successful. Efforts to devise suitable courses of general mathematics for grades above nine have met with little success.**

Assuming, with Betz (3), that organization is necessary in the teaching of mathematics, it may be inquired what effect various types of organization have upon the teaching and learning of mathematics. A direct experiment was performed by Furst (13) to discover whether the organization of learning experiences had any effect upon the organization of learning outcomes. This study, carried on for two years, compared the patterns of learning displayed by 63 students taught by the usual high school method with those of 60 students taught by the non-compartmentalized method of the University of Chicago. The members of the groups were 11th and 12th year students of high intelligence and ability. Three very significant conclusions ensued:

a. There was a clear-cut relationship in mathematics and in the physical sciences between the pattern of emphases taught and that learned. There was no such relationship in English, humanities and the social sciences, however.
b. Even when a determined effort was made to integrate learning outcomes within a subject field and among different fields, the extent of such integration taking place was limited.

c. In the group which was not taught along subject-matter lines, learning had become compartmentalized anyhow!

It may be concluded, tentatively, in the absence of other evidence, that the pattern of emphases in teaching a subject field of mathematics is reflected in learning outcomes, but that present attempts at integration in presentation do not ensure integration in learning. It may be true that the learning of mathematics takes place along subject-matter lines even if it is taught in some different way. The teacher may, therefore, justifiably question the claims made for so-called "progressive" courses of study in mathematics.

(2) THE FRAMEWORK OF INSTRUCTION

For the purposes of this study, this section will include many items which are difficult to classify but which may be said to characterize the instruction. For example, experiments have dealt with special types of lesson (e.g. the supervised study lesson), with various methods of bringing about learning (e.g. the inductive lesson) and with the results of certain "philosophies" (e.g. pupil-centered instruction).

a. Special types of lesson: The following types of lesson have been examined in the experimental literature: supervised study, contract units and special lessons for improving students' ability to solve verbal problems. Other "types" of lesson, highly extolled or adversely criticised, do not seem to have been made the subject of scientific scrutiny.

Experiments on supervised study have been far from clear in their implications. A small, but careful, study by Hunziker and Douglass (19) on two matched groups in geometry and two in algebra used the daily recitation method in the control classes and a supervised study unit plan in the experimental classes. In the experimental group, one to three days were spent in preparation for the unit, then many days in study. Tests revealed no difference in the geometry groups and a difference in favor of daily recitation in the algebra groups. A previous study by Brown and Worthington (5) had shown no difference between supervised study and daily recitations, while a study by Gadske (15) favored the supervised study large unit method.

Some light may be thrown on these results by Schunert's study, performed in 100 Minnesota high schools (35). In this study, it was found that a lesson with (about) 50% supervised study and (about) 50% recitation was significantly advantageous.

A 1928 study performed by Douglass (9), using ten matched pairs of sections, showed that there was no significant difference between recitation followed by supervised study, or supervised study followed by recitation, although the former was somewhat better.

Three experiments on the contract unit method (11, 37, 40) seemed to favor this method in both algebra and geometry, according to summaries in the Education Index.

Some thought has been given to special lessons designed to improve problem-solving ability in algebra. Russell and Holmes (33) used two matched pairs of tenth year algebra classes in an experiment with mimeographed reading exercises. The control group spent the whole period on verbal problems by the traditional method. The experimental group, before doing the same verbal problems, spent about half the period on the prepared reading exercises. It was found that there was no significant difference between the groups except in the algebra reading test, in which the control group was better by a highly significant amount. A similar experiment, by H. C. Johnson (23) in 28 seventh-grade classes, showed that the two groups were equal, at the end of the experiment, in "general learnings" with a very small difference in "problem-solving ability."

In this connection, it is well to take cognizance of Plumlee's experimental work (30) which concludes that

. . . the student who lacks verbal facility is not handicapped to any greater extent on verbal mathematics problems than on non-verbal mathematics problems. The evidence suggests that these two types of problems measure abilities that are highly but not perfectly correlated.

The following tentative conclusions may be drawn concerning the special lessons which have been the subjects of experimental work: (a) It is doubtful that a large unit of supervised study is advantageous. (b) A lesson with part of the time devoted to supervised study and the rest to recitation may be advantageous. (c) The use of supplementary reading materials for verbal problems has not been shown to improve the students' ability to solve verbal problems, and it is likely that the real difficulty is one which deals with all of mathematics rather than with a misunderstanding of the language of mathematics. The teacher should, therefore, view with some skepticism the various special lessons which are advanced as panaceas in education.

b. Certain methods of teaching: Three interesting groups of studies have considered individualized teaching vs. group teaching, inductive vs. deductive development and memorization vs. "verbal rules" vs. "non-verbal clues."

Two studies comparing the traditional group method with individual methods in both algebra and geometry (26, 33) concluded that the results for the experimental and control groups were the same. In Schunert's study (35), it was shown that algebra classes numbering between 20 and 30 students did better work than either smaller or larger classes, whereas class size in geometry classes (within the limits observed) made no difference. It would seem that the individual method of teaching is not better than the group method, but that there may be an optimal group size for successful teaching and learning. This is in accord with field theory which states that group action is a field property.

A splendid experiment by Michael (27) on 15 classes in ninth grade algebra compared the "inductive" method (where the class discovered rules but no generalizations were made) with the "deductive" method (where the teacher gave rules without reasons). There was no difference in "computation" or "aptitudes," but the "deductive" group was significantly better (by a small amount) in "generalization." The topics taught were: positive and negative numbers, algebraic operations and the solution of simple equations.

Hewson (18) performed an experiment upon 689 students in the 8th and 11th grades and in college to compare "memorization," "verbal rules" and "non-verbal clues" as methods of learning. He found that all three methods were reasonably effective and that there was no significant difference in favor of any one of the three methods in the secondary school. Ulmer (39) investigated 1239 high school pupils in three groups matched in age, intelligence quotient and a "reasoning test"

. . . to discover the effect of teaching geometry to cultivate reflective thinking.

Like many experiments of this type, the experimental group excelled in the home-made "reasoning test," but there was no comparison in regular geometry content

The following tentative conclusions may be drawn concerning these special methods which have been investigated: there is no strong evidence to favor inductive vs. deductive, memorization vs. other methods, or individual vs. group teaching so far as achievement in regular mathematics is concerned. The teacher may have a strong suspicion that learning takes place in the same way no matter which method of presentation is employed. Again, this is "in line" with the field theory viewpoint that learning takes place by insight after perception—and that it does not make much difference how perception was accomplished. On the other hand, an atomistic psychology would seem to predict a close resemblance between the type of stimulus and the type of response.

c. Certain philosophies of management: Whereas all the lessons and methods discussed above involve various philosophies of teaching, there are certain experiments which focus their attention upon the philosophy rather than upon other features of the lessons. Some of Jayne's work (20), to be discussed later, deals with classroom practices which may be categorized as "teacher domination" or "pupil centering." It is shown in that study that the incidence of these items made no difference in the achievement of the pupils. Schunert's experiment (35) investigated the effect of "pupil leadership" upon achievement in algebra and geometry. There was no relationship between the amount of pupil leadership and achievement. Bovard (4) has the following startling remark to make concerning "student interaction:"

> Available experimental evidence indicates that the amount of student-to-student verbal interaction in a classroom has no statistically discriminable effect upon a number of variables, including the learning of objective contents, liking for the teacher, and evaluation of the teacher's performance by students.

Syer (38), in an examination of 239 measures of outcomes from 61 research studies, showed that 150 favored the "pupil-centered" methods and 60 the "conventional," while 69 showed "no difference." He rated only 20 of the 61 studies as good, however.

It may be concluded, tentatively, that there is no overwhelming evidence to show that the teacher's philosophy with respect to "domination" or "socialization" of the classroom has any significant effect upon learning outcomes. Of course, it is possible that the "pupil-centered" method brings about a classroom situation in which better learning could take place; or that "other values" are thereby attained. It is also conceivable that no general rule will fit all classes and teachers. Some classes might perhaps flourish under an authoritative teacher, others not. A reading of the experimental literature shows no real basis to accept any one philosophy.

(3) Motivation of the Lesson

All teachers accept without question the doctrine that lessons must be motivated. Beginning teachers, in particular, exercise themselves greatly concerning methods for motivating the lesson. Among many devices recommended are audio-visual aids, field trips and excursions, the use of "real problems" and the interpolation of mathematical recreations.

Donovan A. Johnson performed an experiment (22) in the teaching of mathematics with films and filmstrips. It involved 27 classes in 12 schools. It was found that there was no general difference between classes taught by the experimental method and those taught by the traditional method.

Clark (8) describes an interesting experiment performed in 9 schools with 335 sixth grade students of the social studies. Four units were to be taught: Egypt, Technology, Transportation and Communication. In one group, there were excursions: to the Egyptian room at the museum; to an industrial plant; to another city by bus and train; and to a telephone exchange. Tests at the ends of the units showed no difference between the experimental groups and the control groups in learning outcomes or in interest in the topics taught.

From time to time, articles appear in the literature decrying the "puzzle problem" in mathematics and recommending "real problems." It is well to recall, at this time, Powell's classic study (32) which showed that students preferred problems which expressed the activity of youth and were satisfied with "puzzle problems." They were not especially concerned with the genuineness or importance of these problems. It is evident that the word, "real," requires a more precise definition in terms of the psychology of the student. Puzzle problems may be "real" whereas adult-real problems may not be.

Porter (31) describes three careful experiments in algebra and plane geometry in which one day per week was set aside

for mathematical recreations in the experimental group. In all three experiments, there were significant differences in favor of the experimental groups, in spite of the fact that in these groups about 20% of the instructional time had been taken from the regular topics.

A motivating device well supported by field theory is the use of goal sheets. A small experiment was performed by Shaw (36) using two equated sections of 24 pupils each in ninth year general mathematics. The experimental group was given goal sheets to estimate its own progress through five units of instruction. A comparison of the achievement of the two groups showed a large and significant difference in favor of the experimental group.

A fascinating study by Park (29) investigated motivating devices from the standpoint of the student—surely a novel idea. In this study, 54 high school students and 93 college students were asked, after various lessons, how they had been motivated. The largest percentage of replies (32%) indicated that the students had reacted to "competition, rewards and prizes." The second largest specific category (19%) showed that students had reacted to the teacher's personality and knowledge of subject matter, rather than the motivation which the teacher thought he used. The third category (18%) was audio-visual aids. The other replies (31%) were distributed in miscellaneous ways: interest in notebook work, in publications, in clubs, in construction, etc. A closer examination of students' replies showed, however, that—no matter what they said—the teacher's personality was the important factor in the motivation of the lesson.

Finally, it is most illuminating to read Celler's statistical study (7) of discipline. He says:

> The practice of using all available equipment and visual aids to embellish and enrich a lesson so as to interest and promote the learning process is closely associated with effective discipline.

It may be concluded, tentatively, that some of the recommended motivating techniques do not guarantee better outcomes in terms of content, although they may ensure better classroom discipline; but that others, such as "recreations," "goal sheets" and "teacher enthusiasm" may be successful. A field theory explanation of the effect of these three successful devices is obvious: recreations tend to "space" the learning and to cause "closure" to take place; goal sheets tend to "bring the goal nearer" in psychological jargon; teacher enthusiasm increases the energy level of the group. As a side-light on these conclusions, this may afford an explanation for the fact that in experiments where the experimenter teaches the experimental group, the latter often "wins" over the control group. Can it be that the teacher's enthusiasm, rather than the experimental variable, has brought about the difference between the groups?

(4) Specific Classroom Practices

A supervisor who criticizes a teacher's performance for the purpose of improving instruction ordinarily makes very specific comments concerning the teacher's classroom practices. There are a few studies concerning the effect of specific practices upon learning outcomes. Concerning these, the 1951 Review of Educational Research (24) has the following to say:

> An interesting group of studies has been carried out to determine the comparative effectiveness of classroom practices. Results, in general, were inconclusive.

However, there is an important study, by Jayne (20), which leads to significant conclusions. This study used 28 social studies classes in the seventh and eighth grades and made a careful analysis of the effect of 84 specific teaching acts on the learning of social studies by the children. Some of the items tested were:

Number of comments by the teacher

Percentage of period during which the teacher spoke

Number of times the teacher said, "Yes," instead of allowing the class to decide upon the correctness of a pupil's answer

Number of pupil participations
Length of pupil participation
Number of questions by the teacher
Number of words per question
Number of routine questions
Number of "recall fact" questions
Number of "prepared fact" questions
Number of "unprepared fact" questions

The following quotation describes the effect of these items and all the other specific items evaluated:

> In this study, no single specific observable teacher act was found whose frequency or percentage of occurrence was always significantly correlated with pupil gain . . . the complexity of the teaching act is so great, with its many varying and shifting factors, that a single observable activity divorced from others may not produce an effect measurable under present conditions . . . it is possible that difference in pupil gain may depend primarily upon varying factors inherent in the pupil or his activities . . . rather than in teaching procedures.

If an analogy may be drawn between social studies and mathematics, the tentative conclusion may be drawn that no specific teaching act (including "good questioning") has been shown to guarantee improved learning on the part of the pupil. This is in perfect harmony with the field theory viewpoint that learning is the product of a total reaction between and among pupil, classmates, teacher and situation—rather than any specific act on the part of the teacher.

(5) The Homework Assignment

In planning instruction, most teachers consider the homework assignment an important part of the teaching and learning process. Recent work in the field of social studies by McGill (25) has cast serious doubt on the value of the homework assignment in that field. Foran and Weber (12), working with matched groups in 7 parochial schools, found the homework group slightly better in arithmetic achievement, but made no statistical study of the results. Anderson (1) compared a matched pair of eighth grade students in English, social studies and mathematics and found the homework group somewhat superior, but not significantly so. Schunert's study (35) showed that in algebra, the amount of homework assigned made no difference, but that regular differentiated assignments were better than other kinds of assignments. In geometry, the amount of homework assigned made no difference.

It may be concluded, tentatively, that there is very little evidence either for or against the homework assignment, but that the differentiation of homework assignments is indicated when assignments are given. Field theory gives no prediction concerning the possible value of homework, but is is clear that under this theory repetition, per se, does not bring about learning. Thus, an assignment which is essentially drill should not advance learning.

(6) Tests as Teaching Devices

Although many teachers use tests as teaching devices, rather than as pure marking devices, little has been done in the field of mathematics to evaluate them. A study by Gable (14) on ninth grade biology students compared matched groups in which one group had daily announced tests whereas the other had frequent unannounced tests. The latter was somewhat better. B. E. Johnson (21) used 55 matched pairs of freshman girls in an experiment on tests, and found that whereas tests improved achievement significantly, the difference between experimental and control groups tended to disappear in one to three months. Schunert (35) found that the frequency of tests in algebra made no difference in achievement, but that in geometry the groups that had tests at least once a week were better than those who had fewer tests.

The evidence does not point to any definite conclusions concerning the usefulness of tests as teaching devices. In fact, several questions arise. Are they useful? If they are, what are the differential effects of various lengths, types or frequencies of tests? Are the effects different in various branches of mathematics? Are the effects, if any, temporary or permanent? According to field theory, it would seem that

tests should raise the energy level of the pupil and make for better perception, provided the pupil was successful ("making the goal approach"); and should diminish achievement when the pupil is unsuccessful ("irradiation effect"). Much more research is needed in this field.

(7) The Importance of the Teacher

Having read this far, the reader may begin to entertain a strong suspicion that it doesn't make much difference what the teacher does in the classroom. With very few exceptions, the valid experiments described have shown insignificant or unimportant differences between the experimental and control groups.

However, the aspects discussed have been, in the main, formal aspects of the lesson, itself. It may very well be that the teacher, rather than the teaching, is the important factor in learning. Dunn (10), examining the achievement of 223 boys and girls in ninth grade algebra, found the teacher factor very significant. Ojemann and Wilkinson (28) observed 88 matched ninth grade pupils and noted that as the teacher's understanding of pupil behavior increased there was a slight, but statistically significant, improvement in pupil achievement and attitudes. Other measures (personality conflicts and pupil adjustment) were inconclusive. Gotham (16) compared teacher personality and teaching efficiency and found a multiple correlation of 0.40 between a composite of three personality test scores and pupil improvement. This would indicate a communality of 16% to 40% between the factors measured by the personality tests and the factors antecedent to good pupil achievement. There is, of course, no way of knowing whether the achievement was good because the personality was good, or whether the teacher had attained a satisfactory adjustment because his results were satisfactory.

Schunert (35) compared the pupil achievement of "easy markers" (who usually failed less than 2%) with that of "hard markers" (who usually failed more than 10%). The algebra students of the easy markers did better than those of the hard markers, but there was no consistent relationship in the geometry classes. In the same study, a variety of teacher factors (background, experience, amount and type of training) were examined with relationship to pupil achievement. No consistent relationships were found.

It may be concluded that the teacher factor plays a strong and significant part in pupil achievement, but that the specific *modus operandi* of this factor is in considerable doubt.

(8) Conclusions

An over-all picture, based upon the meager experimentation of the past 50 years displayed in Part I, would probably be as follows:

1. The teacher may justifiably question the claims made for so-called "progressive" courses of study in mathematics. Actually, the evidence seems to show that organization is necessary, but that the pupils learn along subject-matter lines no matter how much correlation or integration is attempted.
2. The teacher can not depend upon any special type of lesson, such as "supervised study," to guarantee success in teaching and learning.
3. There is no decisive proof that any particular method of teaching (inductive, deductive, individual, group) or any particular philosophy of teaching (teacher-dominated lesson or socialized lesson) will guarantee better results than any other method or philosophy, so far as achievement is concerned.
4. Good motivating devices do not, per se, guarantee good learning outcomes. The evidence seems to show that it may be the teacher who is motivated, and that the students react to his enthusiasm.
5. There seems to be no connection between any specific classroom practices (e.g. type of questions) and the success of the students.
6. The evidence concerning the value of homework assignments is scanty, but there is no proof that homework is valuable.
7. The evidence does not indicate that tests have any lasting beneficial effect upon learning.
8. There is considerable reason to conclude, on the basis of the experimental work displayed, that the teacher "as a whole" is the most important factor in the success of the students. The evidence does not show clearly, however, why this is so.

It will be seen from the above that all available evidence rejects an "atomistic" or "connectionist" theory of teaching and learning. The following hypothesis appears to be consistent with the known facts:

The "good" teacher is an understanding person, of the type who can be said to have "personality." His enthusiasm, rather than his background, experience, or training, causes motivation to take place. The motivating devices may possibly bring about some other reaction (such as a desire to come to school, or a desire to take more mathematics) but these have not as yet been measured. The specific acts of the teacher, and even his philosophy, do not seem to have any direct relationship to the learning outcomes. Instead, it is probable that the whole pattern of classroom activities interacts to bring about worthwhile results.

This hypothesis of a psychology of teaching and learning is clearly identical with "field theory."

II. THE IMPROVEMENT OF MATHEMATICAL EXPERIMENTATION

An examination of the results and experiments described in Part I shows that there is a real need for the improvement of experimentation in mathematics education. Benz (2), writing almost twenty years ago, said:

> It is to be hoped that some day we shall be able to write a systematic discussion of the teaching of high school mathematics from the results of more complete research. With the scientific study of the problem of high school teaching in its present state, it is impossible to write an account of the completed research with any semblance of system. This lack of scientific evidence was noted nearly twenty years ago and still persists to some extent.

The same remarks apply, *a fortiori*, to teaching of mathematics on elementary and collegiate levels.

Perhaps the following comments will clarify the reasons for this dearth of scientific evidence in the past sixty years.

1. An experiment in education involves relatively few subjects. Unlike a physical or chemical experiment performed upon millions of randomly moving particles, the educational experiment numbering more than a few hundred cases is rather rare.
2. A second difficulty derives from the fact that the subjects of these experiments are children, dear to us and important to society. If a physicist predicts that a certain solution will increase in temperature by 4 degrees when a certain experimental substance is added, it is of no concern to him that a molecule, here and there, will have a molecular velocity very much less or very much more than that corresponding to this temperature. The educator must, however, consider the entire range of changes before recommending any method, device or philosophy. If a physician discovered a medicine to cure colds in 4 out of 5 people—but which would kill the fifth—who would recommend it?
3. A third difficulty arises from the fact that people, unlike molecules, are importantly affected by the experiment per se. Thus, in comparing Method A with Method B, the teacher and pupil may react to one or both methods, or to the fact that an experiment is being performed. This is called the "interaction effect."
4. A fourth difficulty is that experimental methods are ordinarily new to the teacher. The effect of practice may alter the results, later on, in either direction.
5. A fifth difficulty is the appalling lack of organization in educational research. This is partly due to the fact that changes in organization, methods and philosophies of teaching are often initiated by people who are utilizing intuition and their own limited experience, rather than competent research. It is partly due to the fact that directors of research continue to accept wordy documents filled with the opinions of other wordy authorities—instead of short and pithy investigations on specific questions. It is partly due to the fact that directors of research, through kindness, permit research documents to be published which are completely worthless from a research standpoint.
6. A sixth difficulty is that there is no real agreement, as yet, on the specific objectives of a subject or a course. It is evident (to a mathematician) that definitions and assumptions must precede theorems.

It would seem advisable to secure more valid evidence for the improvement of mathematics instruction. The following items are intended to suggest a plan for reducing some of the sources of error mentioned above:

1. The specific objectives of mathematics, in terms of knowledges, skills, concepts, etc. should be agreed upon. The part to be played by each branch should be settled, at least temporarily. This is the only

place in scientific research where the intuition and opinions of authorities should be sought.

2. Boards of education and universities should assume leadership in investigating practices which seem to promise accomplishment of these specific objectives. A central agency, preferably a national association of teachers, should clear, approve and co-ordinate these researches in order to prevent undue duplication, ensure proper replication and provide direction.
3. Directors of research should insist that investigations be based upon previous good research, where this exists, and that mere opinions be omitted from the final document. Research papers should be brief—so that people will read them—and should be made available to all professional teachers.
4. Untrained persons should not be permitted to do research. Research in this all-important field should be done by people who are already trained teachers and trained researchers. Research training should include techniques of sampling and of the calculation of the reliability and validity of results. It should be noted that "random sampling"—based upon complete ignorance of the composition of a sample—be avoided in educational research. Random sampling is well suited to large, homogeneous populations, but not to the usual educational experiment. Instead of this, "representative" or "stratified" sampling should be used.
5. Because interaction effects may be larger than experimental deviations, it is recommended that experiments use the analysis of variance technique, wherever possible. In this method, it is possible to calculate the interaction effect. Another possibility is the three-matched-group method, in which one group does not know that an experiment is taking place. The experimenter should not be one of the teachers in the experiment, for obvious reasons.
6. Because new methods may suffer or gain because of sheer novelty, it is recommended that all new methods be practiced in a non-experimental set-up for one semester before the onset of the experiment.

Under present circumstances, experiments in mathematics education serve mainly as counter-examples against obviously incorrect hypotheses. Actually, drawing a theory from thirty or forty experiments is very much like drawing a thousand degree function through thirty or forty ill-defined points.

The tools of research require—not sharpening—but better application.

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HAVE YOUR STUDENTS SEEN?

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- "Automatic Control" by Ernest Nagel
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Questions Used in the 1952 Mathematics Contest Sponsored by the Metropolitan New York Section of the Mathematical Association of America

Held Thursday, May 1, 1952

Time of Contest—1 hour 20 minutes Total Credits 150

Instructions

In each of the fifty exercises enclose the correct answer, including the letter that precedes it, in a box. The following is a sample problem.

If the length of a median of a triangle is one-half the length of the side to which it is drawn, the triangle is:

(a) equilateral $\boxed{\text{(b) right}}$ (c) isosceles (d) acute (e) obtuse

Part I. (2 credits each)

1. If the radius of a circle is a rational number, its area is given by a number which is:
(a) rational (b) irrational (c) integral (d) a perfect square (e) none of these

2. Two high school classes take the same test. One class of 20 students made an average grade of 80%; the other class of 30 students made an average grade of 70%. The average grade for all students in both classes is:
(a) 75% (b) 74% (c) 77% (d) 72% (e) none of these

3. The expression a^3-a^{-3} equals:
(a) $\left(a-\frac{1}{a}\right)\left(a^2+1+\frac{1}{a^2}\right)$ (b) $\left(\frac{1}{a}-a\right)\left(a^2-1+\frac{1}{a^2}\right)$
(c) $\left(a-\frac{1}{a}\right)\left(a^2-2+\frac{1}{a^2}\right)$ (d) $\left(\frac{1}{a}-a\right)\left(\frac{1}{a^2}+1+a^2\right)$ (e) none of these

4. The cost C of sending a parcel post package weighing P pounds, P an integer, is 10 cents for the first pound and 3 cents for each additional pound. The formula for the cost is:
(a) $C=10+3P$ (b) $C=10P+3$ (c) $C=10+3(P-1)$ (d) $C=9+3P$
(e) $C=10P-7$

5. The points (6, 12) and (0, −6) are connected by a straight line. Another point on this line is
(a) (3, 3) (b) (2, 1) (c) (7, 16) (d) (−1, −4) (e) (−3, −8)

6. The difference of the roots of $x^2-7x-9=0$ is:
(a) $+7$ (b) $+\frac{7}{2}$ (c) $+9$ (d) $2\sqrt{85}$ (e) $\sqrt{85}$

7. When simplified, $(x^{-1}+y^{-1})^{-1}$ is equal to:
(a) $x+y$ (b) $\frac{xy}{x+y}$ (c) xy (d) $\frac{1}{xy}$ (e) $\frac{x+y}{xy}$

8. Two equal circles in the same plane cannot have the following number of common tangents.
(a) 1 (b) 2 (c) 3 (d) 4 (e) none of these

9. If $m=\frac{cab}{a-b}$, then b equals:
(a) $\frac{m(a-b)}{ca}$ (b) $\frac{cab-ma}{-m}$ (c) $\frac{1}{1+c}$ (d) $\frac{ma}{m+ca}$ (e) $\frac{m+ca}{ma}$

10. An automobile went up a hill at a speed of 10 miles an hour and down the same distance at a speed of 20 miles an hour. The average speed for the round trip was:
(a) $12\frac{1}{2}$ mph (b) $13\frac{1}{3}$ mph (c) $14\frac{1}{2}$ mph (d) 15 mph (e) none of these

11. If $y=f(x)=\frac{x+2}{x-1}$, then it is incorrect to say:
(a) $x=\frac{y+2}{y-1}$ (b) $f(0)=-2$ (c) $f(1)=0$ (d) $f(-2)=0$ (e) $f(y)=x$

12. The sum to infinity of the terms of an infinite geometric progression is 6. The sum of the first two terms is $4\frac{1}{2}$. The first term of the progression is:
(a) 3 or $1\frac{1}{2}$ (b) 1 (c) $2\frac{1}{2}$ (d) 6 (e) 9 or 3

13. The function x^2+px+q with p and q greater than zero has its minimum value when:
(a) $x=-p$ (b) $x=\frac{p}{2}$ (c) $x=-2p$ (d) $x=\frac{p^2}{4q}$ (e) $x=\frac{-p}{2}$

14. A house and store were sold for $12,000 each. The house was sold at a loss of 20% of the cost, and the store at a gain of 20% of the cost. The entire transaction resulted in:
(a) no loss or gain (b) loss of $1000 (c) gain of $1000 (d) gain of $2000
(e) none of these

15. The sides of a triangle are in the ratio 6:8:9. Then:
(a) the triangle is obtuse (b) the angles are in the ratio 6:8:9 (c) the triangle is acute
(d) the angle opposite the largest side is double the angle opposite the smallest side
(e) none of these

Part II. (3 credits each)

16. If the base of a rectangle is increased by 10% and the area is unchanged, then the altitude is decreased by:
(a) 9% (b) 10% (c) 11% (d) $11\frac{1}{9}$% (e) $9\frac{1}{11}$%

17. A merchant bought some goods at a discount of 20% off the list price. He wants to mark them at such a price that he can give a discount of 20% of the marked price and still make a profit of 20% of the selling price. The per cent of the list price at which he should mark them is:
(a) 20 (b) 100 (c) 125 (d) 80 (e) 120

18. Log $p+\log q=\log(p+q)$ only if:
(a) $p=q=$zero (b) $p=\dfrac{q^2}{1-q}$ (c) $p=q=1$ (d) $p=\dfrac{q}{q-1}$ (e) $p=\dfrac{q}{q+1}$

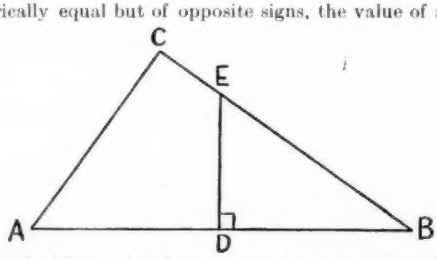
19. Angle B of triangle ABC is trisected by BD and BE which meet AC at D and E respectively. Then:
(a) $\dfrac{AD}{EC}=\dfrac{AE}{DC}$ (b) $\dfrac{AD}{EC}=\dfrac{AB}{BC}$ (c) $\dfrac{AD}{EC}=\dfrac{BD}{BE}$ (d) $\dfrac{AD}{EC}=\dfrac{(AB)(BD)}{(BE)(BC)}$
(e) $\dfrac{AD}{EC}=\dfrac{(AE)(BD)}{(DC)(BE)}$

20. If $\dfrac{x}{y}=\dfrac{3}{4}$, then the incorrect expression in the following is:
(a) $\dfrac{x+y}{y}=\dfrac{7}{4}$ (b) $\dfrac{y}{y-x}=\dfrac{4}{1}$ (c) $\dfrac{x+2y}{x}=\dfrac{11}{3}$ (d) $\dfrac{x}{2y}=\dfrac{3}{8}$ (e) $\dfrac{x-y}{y}=\dfrac{1}{4}$

21. The sides of a regular polygon of n sides, $n>4$, are extended to form a star. The number of degrees at each point of the star is:
(a) $\dfrac{360}{n}$ (b) $\dfrac{(n-4)180}{n}$ (c) $\dfrac{(n-2)180}{n}$ (d) $180-\dfrac{90}{n}$ (e) $\dfrac{180}{n}$

22. On hypotenuse AB of a right triangle ABC a second right triangle ABD is constructed with hypotenuse AB. If $BC=1$, $AC=b$, and $AD=2$, then BD equals:
(a) $\sqrt{b^2+1}$ (b) $\sqrt{b^2-3}$ (c) $\sqrt{b^2+1}+2$ (d) b^2+5 (e) $\sqrt{b^2+3}$

23. If $\dfrac{x^2-bx}{ax-c}=\dfrac{m-1}{m+1}$ has roots which are numerically equal but of opposite signs, the value of m must be:
(a) $\dfrac{a-b}{a+b}$ (b) $\dfrac{a+b}{a-b}$ (c) c (d) $\dfrac{1}{c}$ (e) 1



24. In the figure, it is given that angle $C=90°$, $AD=DB$, $DE\perp AB$, $AB=20$, and $AC=12$. The area of quadrilateral $ADEC$ is:
(a) 75 (b) $58\frac{1}{2}$ (c) 48 (d) $37\frac{1}{2}$
(e) none of these

25. A powderman set a fuse for a blast to take place in 30 seconds. He ran away at a rate of 8 yards per second. Sound travels at the rate of 1080 feet per second. When the powderman heard the blast, he had run approximately:
(a) 200 yd. (b) 352 yd. (c) 300 yd. (d) 245 yd. (e) 512 yd.

26. If $(r+\dfrac{1}{r})^2=3$, then $r^3+\dfrac{1}{r^3}$ equals:
(a) 1 (b) 2 (c) 0 (d) 3 (e) 6

27. The ratio of the perimeter of an equilateral triangle having an altitude equal to the radius of a circle, to the perimeter of an equilateral triangle inscribed in the circle is:
(a) 1:2 (b) 1:3 (c) $1:\sqrt{3}$ (d) $\sqrt{3}:2$ (e) 2:3

28. In the table shown, the formula relating x and y is:
(a) $y=4x-1$ (b) $y=x^3-x^2+x+2$ (c) $y=x^2+x+1$ (d) $y=(x^2+x+1)(x-1)$
(e) none of these

x	1	2	3	4	5
y	3	7	13	21	31

29. In a circle of radius 5 units, CD and AB are perpendicular diameters. A chord CH cutting AB at K is 8 units long. The diameter AB is divided into two segments whose dimensions are:
(a) 1.25, 8.75 (b) 2.75, 7.25 (c) 2, 8 (d) 4, 6 (e) none of these

30. When the sum of the first ten terms of an arithmetic progression is four times the sum of the first five terms, the ratio of the first term to the common difference is:
(a) 1:2 (b) 2:1 (c) 1:4 (d) 4:1 (e) 1:1

31. Given 12 points in a plane no three of which are collinear, the number of lines they determine is:
(a) 24 (b) 54 (c) 120 (d) 66 (e) none of these

32. K takes 30 minutes less time than M to travel a distance of 30 miles. K travels $\frac{1}{3}$ mile per hour faster than M. If x is K's rate of speed in miles per hour, then K's time for the distance is:

(a) $\dfrac{x+\frac{1}{3}}{30}$ (b) $\dfrac{x-\frac{1}{3}}{30}$ (c) $\dfrac{30}{x+\frac{1}{3}}$ (d) $\dfrac{30}{x}$ (e) $\dfrac{x}{30}$

33. A circle and a square have the same perimeter. Then:
(a) their areas are equal (b) the area of the circle is the greater
(c) the area of the square is the greater
(d) the area of the circle is π times the area of the square (e) none of these

34. The price of an article was increased $p\%$. Later the new price was decreased $p\%$. If the last price was one dollar, the original price was

(a) $\dfrac{1-p^2}{200}$ (b) $\dfrac{\sqrt{1-p^2}}{100}$ (c) one dollar (d) $1-\dfrac{p^2}{10{,}000-p^2}$ (e) $\dfrac{10{,}000}{10{,}000-p^2}$

35. When written with a rational denominator, the expression $\dfrac{\sqrt{2}}{\sqrt{2}+\sqrt{3}-\sqrt{5}}$ is equivalent to:

(a) $\dfrac{3+\sqrt{6}+\sqrt{15}}{6}$ (b) $\dfrac{\sqrt{6}-2+\sqrt{10}}{6}$ (c) $\dfrac{2+\sqrt{6}+\sqrt{10}}{10}$ (d) $\dfrac{2+\sqrt{6}-\sqrt{10}}{6}$

(e) none of these

Part III. (4 credits each)

36. To be continuous at $x=-1$, the value of $\dfrac{x^3+1}{x^2-1}$ is taken to be:

(a) -2 (b) 0 (c) $\frac{3}{2}$ (d) ∞ (e) $-\frac{3}{2}$

37. Two equal parallel chords are drawn 8 inches apart in a circle of radius 8 inches. The area of that part of the circle that lies between the chords is:

(a) $21\frac{1}{3}\pi-32\sqrt{3}$ (b) $32\sqrt{3}+21\frac{1}{3}\pi$ (c) $32\sqrt{3}+42\frac{2}{3}\pi$ (d) $16\sqrt{3}+42\frac{2}{3}\pi$ (e) $42\frac{2}{3}\pi$

38. The area of a trapezoidal field is 1400 square yards. Its altitude is 50 yards. Find the two bases, if the number of yards in each base is an integer divisible by 8. The number of solutions to this problem is:
(a) none (b) one (c) two (d) three (e) more than three

39. If the perimeter of a rectangle is p and its diagonal is d, the difference between the length and width of the rectangle is:

(a) $\dfrac{\sqrt{8d^2-p^2}}{2}$ (b) $\dfrac{\sqrt{8d^2+p^2}}{2}$ (c) $\dfrac{\sqrt{6d^2-p^2}}{2}$ (d) $\dfrac{\sqrt{6d^2+p^2}}{2}$ (e) $\dfrac{\sqrt{8d^2-p^2}}{4}$

40. In order to draw a graph of $f(x)=ax^2+bx+c$, a table of values was constructed. These values of the function for a set of equally spaced increasing values of x were 3844, 3969, 4096, 4227, 4356, 4489, 4624, and 4761. The one which is incorrect is:
(a) 4096 (b) 4356 (c) 4489 (d) 4761 (e) none of these

41. Increasing the radius of a cylinder by 6 units increases the volume by y cubic units. Increasing the altitude of the cylinder by 6 units also increases the volume by y cubic units. If the original altitude is 2, then the original radius is:
(a) 2 (b) 4 (c) 6 (d) 6π (e) 8

42. Let D represent a repeating decimal. If P denotes the r figures of D which do not repeat themselves, and Q denotes the s figures which do repeat themselves, then the incorrect expression is:
(a) $D=.PQQQ\cdots$ (b) $10^rD=P.QQQ\cdots$ (c) $10^{r+s}D=PQ.QQQ\cdots$
(d) $10^r(10^s-1)D=Q(P-1)$ (e) $10^r\cdot 10^{2s}D=PQQ.QQQ\cdots$

43. The diameter of a circle is divided into n equal parts. On each part a semi-circle is constructed. As n becomes very large, the sum of the lengths of the arcs of the semi-circles approaches:
(a) equal to the semi-circumference of the original circle
(b) equal to the diameter of the original circle
(c) greater than the diameter but less than the semi-circumference of the original circle
(d) infinite in length (e) greater than the semi-circumference but finite

44. If an integer of two digits is k times the sum of its digits, the number formed by interchanging the digits is the sum of the digits multiplied by:

(a) $(9-k)$ (b) $(10-k)$ (c) $(11-k)$ (d) $(k-1)$ (e) $(k+1)$

45. If a and b are two unequal positive numbers, then:

(a) $\frac{2ab}{a+b} > \sqrt{ab} > \frac{a+b}{2}$ (b) $\sqrt{ab} > \frac{2ab}{a+b} > \frac{a+b}{2}$ (c) $\frac{2ab}{a+b} > \frac{a+b}{2} > \sqrt{ab}$

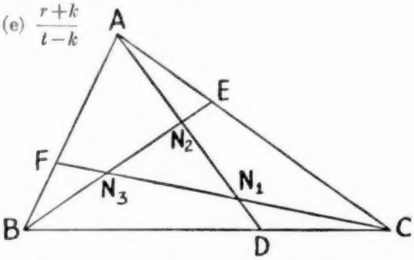
(d) $\frac{a+b}{2} > \frac{2ab}{a+b} > \sqrt{ab}$ (e) $\frac{a+b}{2} > \sqrt{ab} > \frac{2ab}{a+b}$

46. The base of a new rectangle equals the sum of the diagonal and the greater side of a given rectangle, while the altitude of the new rectangle equals the difference of the diagonal and the greater side of the given rectangle. The area of the new rectangle is:
(a) greater than the area of the given rectangle (b) equal to the area of the given rectangle
(c) equal to the area of a square with its side equal to the smaller side of the given rectangle
(d) equal to the area of a square with its side equal to the greater side of the given rectangle
(e) equal to the area of a rectangle whose dimensions are the diagonal and shorter side of the given rectangle

47. In the set of equations $z^x = y^{2x}$, $2^z = 2 \cdot 4^x$, $x+y+z=16$, the integral roots in the order x, y, z are
(a) 3, 4, 9 (b) 9, −5, 12 (c) 12, −5, 9 (d) 4, 3, 9 (e) 4, 9, 3

48. Two cyclists, k miles apart, and starting at the same time, would be together in r hours if they traveled in the same direction, but would pass each other in t hours if they traveled in opposite directions. The ratio of the speed of the faster cyclist to that of the slower is:

(a) $\frac{r+t}{r-t}$ (b) $\frac{r}{r-t}$ (c) $\frac{r+t}{r}$ (d) $\frac{r}{t}$ (e) $\frac{r+k}{t-k}$



49. In the figure CD, AE and BF are one-third of their respective sides. It follows that $AN_2:N_2N_1:N_1D=3:3:1$, and similarly for lines BE and CF. Then the area of triangle $N_1N_2N_3$ is:

(a) $\frac{1}{10}\triangle ABC$ (b) $\frac{1}{9}\triangle ABC$

(c) $\frac{1}{7}\triangle ABC$ (d) $\frac{1}{6}\triangle ABC$

(e) none of these

50. A line initially 1 inch long, grows according to the following law, where the first term is the initial length.

$$1+\tfrac{1}{4}\sqrt{2}+\tfrac{1}{4}+\tfrac{1}{16}\sqrt{2}+\tfrac{1}{16}+\tfrac{1}{64}\sqrt{2}+\tfrac{1}{64}+\cdots$$

If the growth process continued forever, the limit of the length of the line is:

(a) ∞ (b) $\frac{4}{3}$ (c) $\frac{8}{3}$ (d) $\frac{1}{3}(4+\sqrt{2})$ (e) $\frac{2}{3}(4+\sqrt{2})$

William S. Schlauch
1873-1953

William S. Schlauch, emeritus professor in the School of Commerce, Accounts, and Finance of New York University and long-time member of the National Council of Teachers of Mathematics died at his home in Dumont, New Jersey on January 27, 1953. He was born at New Holland, Pennsylvania on February 6, 1873, graduated from the Pennsylvania State Normal at Millersville in 1896, received a B.S. degree from the University of Pennsylvania in 1903 and an M.A. from Columbia University in 1910. From 1905 to 1929 he was a teacher of mathematics and chairman of the department at the High School of Commerce in New York City. In 1929 he joined the staff at New York University as assistant professor in the School of Commerce, Accounts and Finance and held this position until his retirement in 1948. He served on the Board of Directors of the National Council from 1933 to 1936 and as vice president from 1930 to 1931. He was honorary president from 1948 to 1953 and served as one of the associate editors of The Mathematics Teacher from 1935 to 1950. Professor Schlauch contributed papers at the 9th, 18th and 20th annual and the 1942 Christmas meetings of the National Council, served as discussion leader at the 20th and 22nd annual meeting and served on various committees of the Council including the Examinations Committee and the Finance Committee. His contributions to The Mathematics Teacher included the following: "The Use of Calculating Machines in Teaching Arithmetic," January 1940; "A Toast to Teachers of Mathematics," April 1948; and "An Autobiography," November 1948.

General Mathematics and the Core Curriculum

By Ruth Adler *and* Max Peters
Long Island City High School, Queens, New York

The extent to which mathematics can be fused into a core curriculum or correlated with it has been the subject of much speculation and some experimentation.

The fusion of the teaching of arithmetic with other basic elements of a common learnings program has been the recommended procedure on the elementary school level for some time. When arithmetic is learned in functional situations, implicit meanings are not lost behind the symbols used. These observations are equally pertinent on the secondary school level. However, compartmentalized programming has, in the past, made an integrated approach impossible. Furthermore, the necessity of following a sequentially arranged course of study of definite scope makes the integration of algebra or geometry with a core program impractical. On the other hand, in a class in general mathematics there is sufficient flexibility in the selection of materials to make experimentation in correlation with other subjects worth while. The educational literature on mathematics in the core curriculum is very scant. It is hoped, therefore, that this paper, based on one term's experimentation in the correlation of general mathematics with the core curriculum will motivate further experimentation and bring out of hiding work done in other parts of the country along similar lines.

With the introduction of the XG (experimental groups of slow learners) program into the New York City high schools in 1948, the philosophy of integration was accepted for a limited number of students. The successful outcome of these experiments as well as success with similar methods in other parts of the country resulted in the approval by the New York City Board of Superintendents of experimentation with the core program with heterogeneous groups.

Accordingly, in the fall of 1951, experiments with the core curriculum were undertaken at Long Island City and Bryant High Schools. The entire 9th year classes were included in these experiments.

At Long Island City High School the 9th year program is built around an English-Social Studies core which meets daily for a double period with the core teacher. The group remains intact for general science and required art instruction. The fourth major, however, is determined by the individual pupil's program specialization. Thus, academic students take either algebra or a foreign language, and commercial and general students are programmed for General Mathematics 1.

The General Mathematics 1 classes, as was stated previously, are composed of commercial and general students. Commercial students are students of average or above average ability for whom high school is a terminal course. They plan to seek jobs in the business world after their graduation from high school. For the commercial students, General Mathematics 1 is a prerequisite for business arithmetic. The general students, who constitute approximately 50% of the General Mathematics 1 classes, are pupils who, because of IQ, reading and arithmetic scores and previous scholastic work, are considered incapable of pursuing a college-preparatory course. It has been the practice in the New York City elementary school for a number of years to administer standardized achievement and intelligence tests at periodic intervals. During the 8th year, these tests are administered in the March preceding the pupils' graduation in June. The 8th year test results, together with the pupils' general record of academic achievement, and the recommendations of elementary school teachers and principals are the guides for determining whether

pupils shall pursue the academic and commercial courses or the general course.

At Long Island City High School the General Mathematics 1 work was integrated with the core program. The general mathematics teacher met with the other teachers participating in the core experiment for a daily conference period. Correlation with the work of the core teachers was the key factor in selecting activities for the mathematics classroom.

The experiment in the integration of mathematics with the core was handicapped, to some extent, by administrative difficulties. Since the core classes were heterogeneous in nature, not all the pupils in a core class were programmed for general mathematics. Consequently, the general mathematics classes were composed of pupils from as many as four different core classes. While it is true that the general philosophy and units of work were fundamentally the same in all core classes, the specific development varied from teacher to teacher. The topics selected for use in the general mathematics classroom could, therefore, be correlated with the core only in a very general way.

The introduction of some 400 boys and girls into a completely new educational environment quite naturally suggested the first unit of the core, a unit on orientation. Students were taken on getting-acquainted tours of the school building and were introduced to the school's administrators. In the mathematics classroom these tours were used as the springboard for a unit on scale drawings based on floor plans of the school building. Introduction to the school's administrators laid the basis for later discussions with them about various aspects of the school budget. The consideration of the school budget suggested the transition to family budgeting. This was correlated with the second unit of the core—understanding one's self and one's family.

The final unit of the core during its first term was one on getting along with others. In the general mathematics classroom a unit on statistics was undertaken. This unit was oriented towards an appreciation of the basic similarity of all people.

Activities characteristic of core procedures—group and committee work, reports and interviews—were undertaken whenever they fitted in naturally with the work of the classes.

The approach to the teaching of mathematics was flexible. Although the general unit was decided upon in advance by the teachers participating in the experiment, there was freedom from a prescribed course of study. Class discussions at times seemed to veer away from the problem under immediate consideration. However, such discussions were encouraged for two reasons. (1) The mathematics classroom is so often permeated with an atmosphere of tension that discussions, introducing seemingly extraneous elements, tended to develop a more relaxed attitude. (2) Although these "extraneous elements" made no direct contribution to the development of specific mathematical skills, there were positive contributions to the broader area of understandings and appreciations of the world in which we live.

A detailed unit-by-unit discussion, based on careful notes made by the teacher at the end of each school day, will help clarify the general statements made above.

I. Orientation

The orientation tours described in the introduction motivated the unit on scale drawings. The boys and girls made scale drawings of various parts of the school and of their homes. In addition to discovering criteria for selecting a scale, this was an excellent opportunity to develop skills in mensuration. Difficulties in interpreting the graduations on a ruler were numerous. A demonstration ruler was prepared in a core art class to facilitate group instruction in the use of the ruler. The addition of fractional parts of an inch was another stumbling block. This provided an opportunity for drill involving operations with fractions. Emphasis was placed on the con-

crete significance of fractional notation. For many pupils the similarity of the generalized problem, $\frac{1}{4}$ plus $\frac{1}{4}$ equals $\frac{1}{2}$, and $\frac{1}{4}$ inch plus $\frac{1}{4}$ inch equals $\frac{1}{2}$ inch, was a new understanding.

For an evaluation of the unit on scale drawing, it is significant to keep in mind the pupils' comments. With few exceptions, it was the unit they liked least. They stated that they did not understand it. Results of tests on this unit were poor. Since the skills involved in scale drawing are extremely varied (operations with fractions, mensuration, ratios and proportions), they were difficult to master within the period allotted for the unit. Furthermore, the unit took on a formal character which was not in consonance with the spirit of the core. It is likely that the goals in terms of mathematical skills of a unit on scale drawing could have been achieved if the unit were introduced later in the term. This would allow time for laying the necessary basis for work with ratio and proportion.

During this orientation period class officers were elected and bulletin board committees were formed. Each class decided on a name for its bulletin board with some mathematical significance. There were the Squareheads, the Googols (prompted by a discussion of the "largest" number), the Mixed Numbers and the Rulers. Decorative mastheads were prepared in the core art classes. Clippings and classwork related to the unit under immediate consideration were posted by the bulletin board committees throughout the term.

A discussion of the results of a test given at the termination of this unit aroused the pupils' interest in keeping a record of their test results and the class average. This introduced the graph as a device for recording data. The pupils were given freedom in selecting the type of graph they wished to use—line or bar—and in selecting a scale. These graphs became a cumulative record of the pupils' test results.

II. Understanding One's Self and One's Family

An exercise in graphing the average weekly wages of factory workers for the period 1939–1947 produced some valuable discussion. Furthermore, it led directly into the next unit, money management.

The classes were asked to consider possible explanations of the rise in weekly wages from 1939 to 1945, and the sharp drop during the next year. Longer hours during the war years followed by the shortened working day during the period of reconversion brought out the necessity for a more valid basis for comparing wage rates over a period of time—the hourly wage rate. Conversion from weekly to hourly wage rates provided drill in division involving decimal fractions. The end of price control, coupled with the drop in weekly earnings in 1946, led to consideration of what is meant by an "adequate" wage. This provided an opportunity for acquainting the pupils with the Cost of Living Index compiled by the Bureau of Labor Statistics. Cost of living graphs were clipped from the newspapers and posted on the bulletin boards.

Two of the classes had, in the meantime, interviewed the school's administrative assistant and obtained copies of the school budget. The school budget of $12,000 at first seemed staggeringly large. However, when it was translated into cost per pupil per year and when, in particular, the expenditure per pupil per year for textbooks was computed, the boys and girls felt that the budget was inadequate. The circle graph, the type of graph best suited for these data, was considered next.

The remaining two mathematics classes showed little interest in an analysis of the school budget. They preferred to continue their investigations of adequate wage standards. Their judgments of adequate wage standards were remarkably accurate when they were compared with the estimated weekly earnings necessary for minimum standards of health and decency

prepared by the New York State Department of Labor. These two classes derived their budget problem from the budget of a working girl living with her family, as presented in a September 1950 monograph of the New York State Department of Labor. Skills and techniques similar to those described in the unit on the school budget were developed.

New skills developed in connection with this unit were the drawing and measurement of angles. Difficulties with the fundamental arithmetic processes became apparent, particularly in operations involving fractions and in the division of decimals. At this point the process of division was retaught, in an attempt to make it more meaningful. Drill sheets, including a variety of division problems, were distributed. This is but one example of the constant review of fundamental processes throughout the term. Bad habits kept cropping up. Unlearning and reteaching an operation that had originally been learned mechanically is far more difficult than learning the operation correctly in the first place. It is hoped that the new functional arithmetic now being taught in the elementary schools in New York City will provide our boys and girls with a better understanding of arithmetic processes.

The work on budgets was meaningful for the pupils. However, in the attempt to develop comprehensively the finished circle graph from the raw data, there was so much time spent in repititious arithmetic manipulation, that the pupils became restive towards the conclusion of the unit. This falling off of interest could have been avoided by adopting a faster pace. The unit would have had greater significance for the pupils and better correlation with the core if, immediately after the orientation period, the boys and girls had been asked to consider other areas in which budgeting is helpful. The pupils were aware of government budgets. Some suggested family budgeting. Do families budget their expenditures? The pros and cons were discussed with vigor. However, when it was pointed out that a budget did not necessarily have to be a written plan or record and that, in fact, it often was a mental plan of money management, it was generally agreed that most families do have budgets and that unplanned spending with a limited income was an unwise procedure. An evaluation of the pupils' family budgets could have followed logically the analyses of the school and the Department of Labor budgets. However, because of the slow tempo of the original development, any further work in this direction seemed inadvisable.

III. Getting Along with Others

At this time the core classes embarked on their new unit, "Getting Along with Others." In the general mathematics classes the basis was laid for a unit on statistics. After marks in the test given at the end of the preceding unit had been entered on the students' individual graph, the students discussed how they could best compute the class average. In the past, the computation of the class average had been done by the teacher. The pupils suggested the most familiar method—finding the sum of all the marks and then dividing by the total number of pupils. They discovered that this would not be a convenient method of computing the average American income, for example, where the population was very large. This suggested some other method whereby the data could be broken down into 10 or 20 groups with a convenient range within each group. Accordingly, it was decided to group the test marks into 10 groups with a range of 10 points each. Tellers and secretaries were selected to count and record the data. Bar graphs of these frequency distributions were subsequently drawn, and a technique of computing the mean was developed.

The next step in the development of this unit was in the nature of an experiment. Each boy and girl was asked to toss 6 pennies 10 times for their homework assignment. They were to record the number

of times no head, 1 head, 2 heads, etc. appeared. These results were summarized for all four classes. Histograms, based on these summaries, were drawn.

Next, an experiment with a small home-made pin-ball machine was performed in class. Note was taken of the fact that more shot collected in the central slots than in the slots at the ends. The pupils noted further that the graphs of the frequency distributions of their test marks and of the penny-tossing experiment had similar configurations. Was this clustering around the center accidental? A personal data sheet on which the pupils were to record sex, height, weight and shoe size was then distributed. The information was tallied and frequency distributions and histograms were made. Here again the general resemblance in the histograms was noted.

At this point we made a brief excursion into the history of statistics, mentioning the work of Sir Francis Galton with the normal distribution curve. We discussed the meaning of the term, "average," and saw that the arithmetic mean, the average with which the boys and girls were already familiar, does not always give a satisfactory picture of a distribution. This was made especially clear when statistics on average income were presented. The per capita income in New York State for the year 1950 was $2,000. Did this give a true picture of the standard of living in New York State? The pupils commented that this figure included very high incomes. What measure, then, would give a truer picture of the purchasing power of the large majority of people? The concepts of the median and mode were accordingly introduced. Techniques for computing the three measures of central tendency were developed.

The classes next considered the significance of deviations from the average. Although the average shoe size of the girls was $6\frac{1}{2}$, the range extended from 3 to 9. Were the girls at the extremes anomalies? Some said they were. After discussion, it was agreed that factors such as height, weight, shoe size and hand breadth were influenced by inherited body structure.

Valuable correlation with the theme of the core was achieved through an analysis of available statistics on intelligence, body measurements and blood characteristics of various racial groups. The presentation of objective data about the essential similarity of the races of mankind provided an excellent supplementation of the core theme, getting along with others.

The goal in terms of mathematical skills was continued in the form of a unit on formulas. The formulas were always considered in terms of their meaning, although the individual formulas were not interrelated. The evolution of a formula from a statement to its symbolic representation was worked out for perimeters, areas, and discounts. The use of the coefficient and exponent in these symbolic abbreviations was taught. The significance of the sequence of operations in formula substitution was emphasized. It is worthwhile pointing out that the classes derived a good deal of satisfaction from this unit. The satisfaction of following through to successful completion of a small unit of work should not be overlooked. The work with formulas was not correlated with any aspect of the core. The immediate goal was the achievement of specific skills. The pleasure of achieving these skills made this unit worthwhile.

Although this unit was more formal than any we had yet undertaken, in that the formulas were not interrelated and the materials were not tied directly to the work of the core, there were opportunities for discussions and reports. The formula for the conversion of Fahrenheit to centigrade temperatures led to a discussion of the development of standard units of measurement and the criteria for workable units. The advantages of a system of measurement based on groups of ten were made apparent through computations involving both English and metric measurements.

The formula, $s=\frac{1}{2}gt^2$, for the distance through which a freely falling body drops had, as its immediate goal, the meaning of exponential notation. The significance of the formula was brought out through discussion. If a filled mail bag and a pound weight were dropped simultaneously from an airplane (neglecting air resistance), which would land first? With but one exception the pupils agreed that the heavier object would land first. A simple experiment with two pieces of chalk of unequal size indicated that this was apparently not so. It was noted that weight did not enter into the formula at all. Galileo and his famous experiments were mentioned. Several students volunteered to make reports on Galileo and his work.

Our final unit was introduced the week before the Christmas holidays. A large mail-order house had supplied the teacher with 120 catalogues. Our objective was planning a mail order purchase. The boys and girls were given complete freedom in planning. In most cases, they elected to work in groups. Their "purchasing," rather than being aimless, was usually planned around a specific theme. Some furnished a living room or kitchen, others purchased a teen-age girl's or a child's wardrobe. One boy equipped an office. Most of the boys purchased equipment for a hunting trip, including guns, appropriate clothing, tents and sleeping bags. Some did their Christmas shopping.

Specific mathematical skills used in this unit included work with denominate numbers, per cents, mensuration and areas. The unit was rich in its potential for supplying the pupils with useful general information. Discussions arose about installment buying, deceptive advertising, rural mail delivery, and modes of transportation. Pupils whose relatives worked for the U. S. Post Office and Railway Express Agency (which has a large office near the school) reported on the functions of these agencies.

We terminated this unit with the writing of checks in payment for the purchases. Most of the pupils believed that only rich people had checking accounts. It came as a surprise that a special checking account is cheaper than a money order for payments by mail and is a sound investment when the head of the family is paid on a monthly or semimonthly basis.

The possibilities for integration of a mail-order unit are extensive. In our own experience, we were hampered by the fact that the catalogues did not arrive until the term was almost over. However, if the catalogues are available so that the unit can be introduced at the most opportune time, it has the following possibilities:

1. Integration with family living.
2. Consumer education through consideration of quality, best buys, and advertising claims.
3. Transportation, through bringing the merchandise to the consumer. The interdependence of people in a technological civilization could be worked in at this point through an examination of the distribution of manufactured goods from manufacturer to consumer.
4. Advantages and disadvantages of credit buying, with its relationship to family income and money management.
5. Specific mathematical skills.

Conclusion

This experiment in the correlation of general mathematics with the core curriculum was definitely worth-while. There are positive lessons to be learned from this experience among which are the following:

1. Although there was no prescribed course of study, the course had as substantial a mathematical content as the more formal general mathematics course. The specific mathematical concepts and skills covered included fundamental operations, decimals, fractions, per cents, denominate numbers, linear measure, angle measure, graphs, averages and formulas including work with exponents.

2. It is possible to relate significant mathematical topics to core topics, e.g. the units on money management and statistics.

3. It is not necessary to relate all

mathematical topics to the core. It should not be overlooked that, in addition to the other interests of adolescents around which the core is built, they have an interest in learning as such. In the unit on formulas, wholly unrelated to the core, the classes derived genuine satisfaction and a sense of accomplishment from learning something new.

4. The opposite danger of work that is too diffuse, as in the unit on scale drawing, or in excessive drill to the point of boredom, as in the unit on circle graphs, should be avoided.

5. If the mathematics class were composed of only one core group, instead of a few pupils from several core groups, it would have been possible to achieve closer correlation with the core, art and general science classes.

6. Many of the worthwhile activities engaged in (the mail order purchase unit, for example) can be carried out in any general mathematics class, whether or not it is associated with a core program. This suggests an important value of the core experiment. Because of the flexibility of approach, it encourages a freer selection of materials and a bolder variation of methods. As a result it increases the likelihood of discovering and developing new and better methods that can be applied in any classroom. These will be a lasting gain for the teaching of mathematics, whether or not the core program becomes a permanent feature of the high schools.

MEMBERSHIP REPORT
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
February 2, 1953

State	*Individual*	*Institutional*	*Total*
Alabama	83	32	115
Arizona	27	13	40
Arkansas	96	16	112
California	252	196	448
Colorado	101	21	122
Connecticut	113	38	151
Delaware	35	4	39
District of Columbia	122	13	135
Florida	165	43	208
Georgia	84	32	116
Idaho	8	6	14
Illinois	686	113	799
Indiana	261	53	314
Iowa	174	46	220
Kansas	186	24	210
Kentucky	53	24	77
Louisiana	146	32	178
Maine	28	9	37
Maryland	152	30	182
Massachusetts	232	53	285
Michigan	239	81	320
Minnesota	176	53	229
Mississippi	57	22	79
Missouri	131	41	172
Montana	28	9	37
Nebraska	134	19	153
Nevada	2	3	5
New Hampshire	31	10	41
New Jersey	249	71	320
New Mexico	33	19	52
New York	533	188	721
North Carolina	122	43	165
North Dakota	22	11	33
Ohio	342	73	415
Oklahoma	102	48	150
Oregon	48	17	65
Pennsylvania	426	160	586
Rhode Island	31	8	39
South Carolina	60	36	96
South Dakota	18	8	26
Tennessee	100	41	141
Texas	299	125	424
Utah	14	10	24
Vermont	16	7	23
Virginia	192	51	243
Washington	58	38	96
West Virginia	68	12	80
Wisconsin	229	51	280
Wyoming	26	6	32
TOTALS	6790	2059	8849
U. S. Possessions	21	13	34
Canada	80	50	130
Foreign	80	86	166
GRAND TOTALS	6971	2208	9179

M. H. AHRENDT, *Executive Secretary*

Thirty-First Annual Meeting of The National Council of Teachers of Mathematics

The Ambassador, Atlantic City, New Jersey
April 8, 9, 10, 11, 1953

The National Council of Teachers of Mathematics will hold its thirty-first annual meeting at the Ambassador Hotel in Atlantic City, New Jersey, on April 8th through the 11th, with the Association of Mathematics Teachers of New Jersey as the host organization. A full and varied program of exceptional range and interest has been prepared for this meeting, and lectures by distinguished speakers on timely topics in pure and applied mathematics are scheduled for each day. Four continuity Discussion Groups, each meeting in three sessions planned as a continuous sequence, will assemble in the hotel's spacious and attractive rooms. The material to be discussed will include methods of teaching, laboratory techniques, and trends and topics of interest from the elementary level through the college level.

Among the scheduled lectures of unusual general interest are the following: The Contributions the U. S. Office of Education May Make to Mathematics (Kenneth E. Brown, Specialist for Mathematics, Office of Education, Washington, D. C.), Teacher Encouragement (W. W. Rankin, Duke University, Durham, N. C.), The Twenty-First Yearbook (Howard F. Fehr, Editor, Teachers College, Columbia University, New York), The Future of Mathematics Education (William D. Reeve, New York, N. Y.), Reason and Rule in Arithmetic and Algebra (Ralph Beatley, Graduate School of Education, Harvard University), Statistical Quality Control (Paul C. Clifford, State Teachers College, Montclair, N. J., and Ellis R. Ott and Mason Wescott, Rutgers University, New Brunswick, N. J.). In addition to these, lectures of more particularized interest will be given by outstanding specialists in the four levels of mathematics teaching.

Panel Discussions, involving the participation of prominent authorities in mathematics education, will be held on the following topics: Meeting the Needs of the Gifted Student, Integration of Subject Matter in the Conventional Course in High School Mathematics, The Need for

Panorama of Boardwalk and Hotel, Atlantic City.

Headquarters Hotel—The Ambassador.

Better Articulation between Secondary Schools and Colleges to Provide Adequate Mathematical Training for Future Engineers, Mathematicians and Scientists.

The general theme of the Annual Meeting, on which lectures and discussions will converge, is Increasing Student Participation in Mathematics Classes. The complete program for the meeting appeared in the February number of THE MATHEMATICS TEACHER.

The charm and attractions of Atlantic City hardly need advertisement at this date: they have for many years made this city one of the world's most popular and famous seashore resorts. Especially celebrated and universally admired is the superb, traffic-free Boardwalk stretching the entire length of the island on which Atlantic City is built, nearly eight miles, along the broad sandy beach. For those who wish transportation along the Boardwalk, a unique mode of conveyance is provided: the Rolling Chair, an original invention dating back to the '80's. Another typical Atlantic City method of transportation is the Jitney Bus. The Jitneys operate on Pacific Avenue, one block north of the Boardwalk, and they still take you the entire length of the city for ten cents.

Further notable attractions of this resort city are its splendid parkways, its restaurants, some four hundred of them, catering to every variety of gourmet taste, and its magnificent hotels, of which the Ambassador is one of the finest. Visitors cannot fail to find a wealth of opportunities for recreation and pleasant association in a delightful atmosphere of congeniality.

The registration fee for the Annual Meeting is \$0.50 for members of the National Council and \$1.50 for nonmembers and visitors. For further details see the notice and registration blank in the February issue of THE MATHEMATICS TEACHER or write to Miss Gladys E. Estabrook, Cranford High School, Cranford, New Jersey. You will want to make

advance reservations for the Convention Luncheon ($3.20) to be held on Saturday and for the Banquet, a New Jersey Shore Dinner ($4.60), scheduled for Friday night. Requests should be accompanied by check or money order. All orders received before March 25th will be acknowledged by return mail.

Daily rates, European plan, for rooms with private baths at the Ambassador are as follows: single, one person, $6, $8, $10, $12, $14; double, two persons, $8, $10, $12, $14, $16, $18; parlor, two doubles, two baths, $32, $42, $54. Additional charge for third occupant of room: $3.00 daily. Applications for room reservations should be sent directly to the Ambassador, the Boardwalk, Atlantic City, New Jersey.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Membership Record

Mary C. Rogers, *Roosevelt Junior High School, Westfield, N. J.*

100% Schools—as of December 10, 1952

1.	Freeport, Illinois	Freeport High School
2.	Kankakee, Illinois	Senior High School
3.	Carthage, Missouri	Carthage High School
4.	Phillipsburg, Montana	Granite County High School
5.	Highland Park, New Jersey	Highland Park High School
6.	Newark, New Jersey	State Teachers College
7.	Sussex, New Jersey	Sussex High School
8.	Swedesboro, New Jersey	Swedesboro High School
9.	Trenton, New Jersey	State Teachers College
10.	Wood-Ridge, New Jersey	Wood-Ridge High School
11.	Richmond, Virginia	John Marshall High School
12.	Washington, D. C.	Anacostia High School
13.	Washington, D. C.	Cardoza High School
14.	Washington, D. C.	Cooledge High School
15.	Washington, D. C.	Eastern Junior-Senior High School
16.	Washington, D. C.	Gordon Junior High School
17.	Washington, D. C.	McKinley High School
18.	Washington, D. C.	Kelly Miller Junior High School
19.	Washington, D. C.	Garnet Patterson Junior High School
20.	Washington, D. C.	Paul Junior High School
21.	Washington, D. C.	Phelps Vocational High School
22.	Washington, D. C.	Roosevelt High School
23.	Washington, D. C.	Stuart Junior High School
24.	Washington, D. C.	M. M. Washington Vocational High School
25.	Washington, D. C.	Western High School
26.	Green Bay, Wisconsin	West High School
27.	Washington, D. C.	Capitol Page School

"All but one" Schools—as of December 10, 1952

1.	Convent Station, New Jersey	College of St. Elizabeth
2.	Elizabeth, New Jersey	Battin High School
3.	Jersey City, New Jersey	State Teachers College
4.	Millville, New Jersey	Memorial High School
5.	Mount Holly, New Jersey	Regional High School
6.	Red Bank, New Jersey	Senior High School
7.	Washington, D. C.	Dunbar High School
8.	Washington, D. C.	Spingarn High School
9.	Washington, D. C.	Woodrow Wilson High School

The President's Page

NATIONAL COUNCIL MEETINGS

The program of the Thirty-First Annual Meeting of The National Council of Teachers of Mathematics to be held at the Ambassador Hotel in Atlantic City, April 8, 9, 10 and 11, 1953 was published in The Mathematics Teacher last month. One hundred forty persons will have a part on the program, and in addition nearly 150 people are serving on the committees on local arrangements. Program participants come from 33 states. The fact that three persons from California and one from the state of Washington are willing to travel 3000 miles to make their direct contributions to the improvement of mathematics education is testimony to the importance of our programs and the worth that is attached to participation in them by teachers everywhere.

Meetings of the Council are among our most important activities. They provide an opportunity for wide participation in the discussion of problems in mathematics education at all levels of instruction. The program planned for Atlantic City which is organized like programs of other annual meetings, includes addresses on curriculum and instruction in mathematics, mathematical topics, history of mathematics, professional opportunities in mathematics, and applications of mathematics; panel discussions on provisions for the gifted student, integration of subject matter, reexamination of general mathematics, problems of the small high school, articulation between secondary schools and colleges, and problems of mathematics supervisors; shorter papers and discussion groups on a wide variety of topics important in mathematics education; laboratories for the preparation of teaching aids; demonstration lessons; visits to Atlantic City classrooms; and the showing of mathematics films.

Teachers who attend will obtain new ideas on mathematics and the teaching of mathematics, new perspectives on the total mathematics programs of their schools, inspiration from association with old and new friends, and a new sense of the importance of the work they are doing. They will also by their attendance, contribute to and have a part in the work of an important national organization devoted to better schools for America.

A National Council program offers so many opportunities that careful selections of sections to be attended have to be made. Each person who attends will select those sections in which the topics are related to his most urgent needs as he sees them and in which the speakers are those that he wants most to hear. He should also try to include a broad coverage of topics and by all means should attend some sections at a level of instruction other than his own. For example, the senior high school teacher should attend one or more sections at the elementary, junior high school and college levels if he can possibly do so. The schedule of sessions attended by any one person should include also some selected for general interest and purposes of general education. If each one who attends selects several of the sessions in which he will have an opportunity to take an active part in the discussions he will find himself well repaid and also will have contributed to the value of the meeting for others.

Our present schedule calls for four meetings of the National Council each year. The Annual Meeting at which the Annual Business Meeting and the Annual Delegate Assembly are held is scheduled each year during the spring. The other meetings for any year are the Summer Meeting, the Christmas Meeting, and the one-day meeting held in conjunction with the National Education Association. Most mem-

(Continued on page 184)

The National Council Affiliated Groups

By William A. Gager, *Regional Director Southeastern States*
The University of Florida, Gainesville, Florida

Affiliated Groups and the 1953 Annual Meeting

The program of the Thirty-first Annual Meeting of the National Council of Teachers of Mathematics which will be held at the Ambassador Hotel in Atlantic City, April 8–11, 1953, lists these meetings of special interest to the Affiliated Groups:

Thursday, April 9, 8:00–9:00 A.M. Breakfast for Delegates in the Embassy Room. (Get your ticket for this breakfast when you register.) 9:00–11:00 A.M. First Meeting of Delegate Assembly in Room 125.

Friday, April 10, 8:00–10:00 A.M. Second Session of the Delegate Assembly in Room 125.

Saturday April 11, 2:30–4:00 P.M. Panel Discussion sponsored by the Committee on Affiliated Groups.

In earlier communications it was announced that there would be a luncheon for delegates followed by a session of the Delegate Assembly. This has been changed and there *will not* be a delegates' luncheon or any afternoon sessions of the Delegate Assembly. This year's social "get together" for the delegates will be the breakfast at 8:00 Thursday morning. This informal "get acquainted" breakfast should prove to be an excellent fore-runner to the more formal business of the Assembly. Talking over the breakfast table should make the participation in the business that follows more spontaneous and influence a greater per cent of the delegates to participate.

Because the National Council program is so long, and so broad in its scope, it was thought best not to encourage local Affiliated Groups to sponsor programs at the Atlantic City Meeting. However, since it was the vote of the Third Delegate Assembly to have Affiliated Groups sponsor programs, this practice will be continued at summer and Christmas meetings.

The Panel Discussion sponsored by the Committee on Affiliated Groups, which meets in Room 125 Saturday 2:30–4:00, should prove to be a very profitable meeting. The panel, composed of Kenneth E. Brown, Myrl H. Ahrendt, Robert Pingry, John Schacht, with Donovan A. Johnson as chairman, will discuss the topic "*Meeting the Needs of the Gifted Student.*" The chairman and several of the other men on this panel, along with about 150 other educators, recently spent three days at the Office of Education in Washington studying this kindred problem: "How to Identify and Provide for the Gifted Student?" This means that Mr. Johnson and his Panel will have the latest and the best information now available on this important topic. Don't leave Atlantic City without permitting this Panel to share its information and experiences with the gifted student with you.

The Delegate Assembly Sessions will be the place to meet many of the National Council leaders. Special guests will be John Mayor, President of the National Council; Myrl Ahrendt, Executive Secretary; and Kenneth Brown, Specialist in Mathematics from the U. S. Office of Education. Other National Council leaders who have accepted invitations to appear before the Delegate Assembly are E. H. C. Hildebrandt, Editor of The Mathematics Teacher; H. B. Risinger, Chairman of the Speakers' Bureau; and Miss Madeline Messner, who is in charge of the Traveling Exhibit. Tentative plans also call for the presence of Henry Syer, Chairman of the Committee on Publications of Currect Interest; F. Lynwood

Wren, Chairman of the Yearbook Planning Committee; Henry Van Engen, Chairman of the Committee on Research; and Houston Karnes, Chairman of the Committee on Contests and Talent Search.

At the Delegate Assembly you will also have opportunity to become better acquainted with your four Regional Directors of Affiliated Groups:

Jackson Adkins from the Phillips Exeter Academy, Exeter, New Hampshire who represents the 12 northeastern states;

Donovan Johnson from the University of Minnesota High School, Minneapolis, Minnesota, who represents the 13 northwestern states;

Ida Mae Bernhard from the Texas Education Agency, Austin, Texas, who represents the 12 southwestern states; and

William A. Gager from the University of Florida, Gainesville, Florida, who represents the 12 southeastern states.

Your Regional Directors of Affiliated Groups are vitally interested in the problems and in the development of each Affiliated Group. Get to know them and make good use of their willingness to serve your organization.

As is stated in its reason for existing, the basic purpose of the National Council of Teachers of Mathematics is to improve the teaching of mathematics and mathematical education at all levels of instruction. To accomplish this purpose the individual teacher must be reached. On the other hand each teacher must have an avenue for expression available to him. This avenue is now being provided by the practice of each Affiliated Group sending a delegate to the Delegate Assembly. Those who have observed how it works know that, each year, the outcomes of this Delegate Assembly, which are composite expressions of the thinking of the individual teachers back home, are becoming more useful and more helpful to those officers of the National Council who have to make the final decisions on policies and programs.

Because of the importance of the Delegate Assembly as a clearing house for National Council policy making, each Affiliated Group, that has not already done so, is urged to select immediately its delegate to the Atlantic City Delegate Assembly. Send the name of your delegate to Mary C. Rogers, Chairman of the Committee on Affiliated Groups, 307 Prospect Street, Westfield, New Jersey.

If the delegate appointed from each Affiliated Group would take the time to discuss the agenda for the 1953 Delegate Assembly with the other officers and members from that group, it would prove to be extremely valuable. This procedure would tend to give a better representation of the group. Then, too, it could uncover other important items that should be placed before the Delegate Assembly. (For the Delegate Assembly agenda see the January 1953 issue of The Mathematics Teacher or Affiliated Group Newsletter Volume III, No. 1, November 10, 1952.)

Affiliated Groups Having 75% or More Memberships in National Council

Some groups that may belong under the above classification have not renewed their affiliation as yet. This is probably because the date for annual renewal of affiliation does not expire until December 31, 1952. For such groups credit cannot be given at this writing. However, one group has been heard from that deserves very special mention. The Louisiana-Mississippi Affiliated Group has 100% of its members also members of the National Council. No matter how one looks at such a record it is going to be hard to beat. Yet if it can't be beat it can be matched. How about it? Who will be the others with 100%?

Any Affiliated Group that has 75% or more of its members also members of the National Council will have its annual dues waived. What a privilege and help to

have at least 75% of an Affiliated Group profiting from THE MATHEMATICS TEACHER, one of the best subject matter journals available in any area. Besides each person represented by this 75% is eligible to take part in all the activities of the National Council.

NEW AND PROSPECTIVE AFFILIATED GROUPS

It is most encouraging to know that there are 12 different groups working toward affiliation at this writing. Before this article makes its appearance, it is expected these 12 groups will have completed all affiliation requirements, and will have elected their delegates for the Atlantic City Delegate Assembly.

Congratulations to the following groups which have recently completed their affiliations: The Wyoming Council of Teachers of Mathematics, The Mathematics Club of Greater Cincinnati, and The Delaware Mathematics Association. The completion of the affiliation is a privilege for all parties concerned. It is also a responsibility. As an Affiliated Group you share that great responsibility of promoting the stated purpose of the National Council. This in turn will promote the welfare of each individual mathematics teacher.

President's Page

(*Continued from page 181*)

bers, of course, can not attend all of the meetings of any year, but with a good geographical distribution of the meetings every member should find it possible to attend at least one meeting each year. The total registration at the four meetings of 1953 should exceed 2200 if expectations based on past experiences are met. Since a considerable number of non-members are registered at each meeting, this estimate is much smaller than we should be able to expect. By attending ourselves, and encouraging our friends to attend, we can all help to exceed this estimate in 1953!

JOHN R. MAYOR,
President

The schedule of meetings of the National Council of Teachers of Mathematics for the next two years is:

April 8–11, 1953	Ambassador Hotel, Atlantic City, N. J.
June 29, 1953 (with the N.E.A.)	Miami Beach, Florida
August 24–26, 1953	Western Michigan College of Education, Kalamazoo
Christmas 1953	University of California in Los Angeles
April 1954	Sheraton-Gibson Hotel, Cincinnati, Ohio
Summer 1954 (with the N.E.A.)	New York City
Summer 1954	University of Washington, Seattle

HAVE YOU SEEN?

In *The Journal of Business Education*, December 1952
"The 3 R's are Essential" by Alfred E. Waller
In *American Journal of Physics*, December 1952
"Let's Be More Specific" (in teaching Calculus and Physics) by Donald M. Bennett
In *The Clearing House*, December 1952
"Solid Geometry with Models: No Failures in 4 Years" by Willa W. Corbitt

MATHEMATICAL RECREATIONS

Edited by Aaron Bakst
135-12 77th Avenue, Flushing 67, N. Y.

In the January 1953 issue of The Mathematics Teacher this department offered some observations concerning some recreational activities which may be associated with certain specific properties of the principles of system of numeration. Generally, the properties of systems of numeration are not included in the scope of mathematical instruction in the secondary schools. This is unfortunate if not deplorable. Teachers, teachers of teachers, textbook authors, proponents of considerations of pedagogical theories in mathematical education, all of them proclaim their allegiance to the principle that proper and interesting illustrative material is a *sine qua non* of good classroom instruction. The relation between these proclamations and actual practice may be non-linearly inversely proportional. It is very commendable to make proposals. But it is distressing when *theory* is left at the church waiting for *practice*. The indecision of the hapless bridegroom need not be classified as jilting. More often than not it is a case of lack of courage .... It may be a case of lack of information .... It can also be a case of mathematical instruction in the school taking a terrific beating from those who would channel the teaching of mathematics into directions which would reduce mathematics to a useless non-essential. For those who treat mathematics in this manner and for those interested in mathematics teaching and who meekly accept and subscribe to the false prophesies, this department offers a passage from Luke, 23: 34. For those who are eager to wed theory and practice, Matthew, 7: 7 is suggested. All those, who are urged to divert mathematical instruction to the new paths, should first consult Matthew, 24:24 and Mark, 13:22.

This department does not propose to convey the impression that the topic on systems of numeration is the panacea for all the ills of mathematical instruction. The aim and goal of the present discussion is an illustration of what one may accomplish both in the classroom and in a recreational activity with a single isolated field of elementary mathematics. Any other field may offer similar opportunities. Any other topic may be exploited for the desired purposes with equal success.

Recreational activities with systems of numeration are not mere puzzlers. The mathematical bases for such recreations are deeply rooted in fundamental properties which need be thoroughly understood. However, these fundamental properties are so analogous to the properties of the decimal system of numeration that the generalization of the numerical value of the base is a procedure which is usually provided for in the fundamental objectives of mathematical instruction on the secondary school level. Thus, the transition from the base 10 to any other base should not offer many undue difficulties. If there may be a difficulty, it would be associated with the necessity of concentrating one's attention on the fact that a new numerical base is present.

On the other hand, consider the opportunities for practice in arithmetic which new systems of numeration offer. For example, the addition of 3163 and 4512, when both are written in systems other than the decimal, requires close attention to the value of the base. Such addition may call forth powers of concentration and analysis. Suppose that the following problem is offered for solution:

$$3163+4152=10345,$$

determine the base of the system of nu-

meration. The clue is quite obvious since $3+4=7$ and in this sum $3+4=10$. Thus, the numbers are written in the "seven-system." Suppose, however, that the sum is $3163+4152=7325$. The clue is in the addition of 6 and 5. The sum of 6 and 5 is 11. In this case, however, we have 2, and 1 was carried to the place on the left. Thus, these two numbers are written in the "nine-system." Similar problems in other arithmetic operations may be devised by the teacher.

The simplest system of numeration is the binary system (or the "two-system") in which the only two digits are "0" and "1." This system of numeration is very important in the construction of certain electronic digital computers.[1] This system is also the basis for many recreational activities.[2]

Consider, for example, the so-called "Russian Peasants' Multiplication." Actually, this is a multiplication method which is found in the Rhind Papyrus. This method of multiplication is very old. It is a method which for some unknown reason is encountered in different countries and among many different nationalities. The fundamental principle of this method of multiplication is rooted in the binary system of numeration. The main procedures of this method consist of halving one factor (multiplicand) and doubling the other factor (multiplier). Let us consider the following example: $16\cdot 23$. The multiplication is performed as follows:

Halving	Doubling
16	23
8	46
4	92
2	184
1	368

[1] The reader will find it very instructive to read the discussion of the uses of the binary system in electronic digital computers in the book by Edmund C. Berkeley, *Giant Brains or Machines That Think*. New York: John Wiley & Sons, Inc., 1949.

[2] The material presented here is part of the subject matter on systems of numeration which will appear in a forthcoming book on *Mathematical Pastimes and Games* by Aaron Bakst (New York: D. Van Nostrand Co., Inc.). All rights reserved.

Thus $16\cdot 23=368$.

If the multiplicand is odd and it is not a power of 2, the multiplication process is performed as follows. Let us multiply 19 by 35.

Halving	Doubling	Summing
19	35	35
9	70	70
4	140	...
2	280	...
1	560	560
		665

There is a definite reason for adding only certain doubled numbers. This is explainable in terms of the binary system of numeration. The decimal number 19 is translated into the binary system as 10011. Thus

$$19=1\cdot 2^4+1\cdot 2+1,$$

and

$$19\cdot 35=(1\cdot 2^4+1\cdot 2+1)\cdot 35$$
$$=35\cdot 1+35\cdot 2+35\cdot 2^4.$$

This indicates that the doubled multipliers must be multiplied by the remainders which occur in the halvings of their corresponding multiplicands. Whenever a multiplicand is even, the reminder is "0" and thus, in the case of even multiplicands, the multipliers which correspond to them are disregarded and not added. Schematically this may be illustrated for the following multiplication

Halving	Remainder	Doubling	Adding
39	1	43	43
19	1	86	86
9	1	172	172
4	0	344	....
2	0	688	....
1	1	1376	1376
			1677

This suggests a simple rule for halving of odd multiplicands. Subtract unity from the multiplicand and divide the difference by 2. Whenever a multiplicand is odd its corresponding multiplier (properly doubled) must be added.

The multiplication process illustrated above may be called "painless multiplication without the multiplication table."

Another interesting property of systems of numeration is related to the fact that systems of numeration are not isolated phenomena. There is an interrelation between various systems of numeration. Let us consider the binary system as a starting point. The first eight numbers of the decimal system of numeration are represented in the binary system as follows:

000	represents	0	100	represents	4
001	represents	1	101	represents	5
010	represents	2	110	represents	6
011	represents	3	111	represents	7

If a binary number is written in terms of triplets, then each triplet may be transcribed in terms of the relations stated above. Thus, for example, the binary number 11,001,010,101,111,000,100,110,101,-011,001,000,111,011 is transcribed as 31,257,046,531,073. The first two digits "11" may be considered as having been written as "011."

The number 31,257,046,531,073 is written in the system of numeration whose base is eight. Such a system of numeration is known as the "eight-system" or the octal system.

By means of the eight relationships given above we may translate an octal number into a binary number without going through much (and unnecessary) arithmetic work.

Consider the relationship between the "three-system" of numeration and the "nine-system" of numeration. In the "three-system" of numeration we have the following expressions for the first nine digits

00	represents	0	11	represents	4
01	represents	1	12	represents	5
02	represents	2	20	represents	6
10	represents	3	21	represents	7
			22	represents	8

If we have a number which is written in the "three-system" of numeration such as, for example, 22121002011021122012, we rewrite it in terms of pairs as follows 22,12,10,02,01,10,21,12,20,12. Then, by means of the above relationships we rewrite it as 8532137565, and this number is written in the "nine-system" of numeration.

There are many other interesting properties of systems of numeration which are not only recreational but instructive as well. The Mayan system of numeration, the Japanese and Chinese abaci are best explained in terms of the properties of systems of numeration.

A mathematical recreation may be considered just as some relaxing pastime. But, under proper organization a mathematical recreation may become a powerful instrument of instruction. It may open vistas in mathematics which otherwise would be closed to the pupils. It may be related to almost all the fields of human activities. It may reveal certain cultural facts which are the common heritage of Mankind. A mathematical recreation is not something which is designed for amusement purposes only, notwithstanding some modern theories of education that the pupil must be "amused." A mathematical recreation brings to the fore those mathematical values which, under the traditional conditions of instruction, are dull and uninteresting. A mathematical recreation will not contribute to the teaching of citizenship, the furtherance of democracy and the attainment of all other intangibles which are not the domain of mathematical instruction. But a mathematical recreation will assist in the development of those mathematical powers which should be the equipment of every individual. Our society has undergone a long and healthy development during the last two centuries, and we need not saddle the mathematics teacher with those tasks which do not belong in the mathematics classroom.

Correction: In the January 1953 issue of THE MATHEMATICS TEACHER this department permitted a few very grave errors to creep in at the bottom of page 56 while presenting the illustration of the principle of testing divisibility. The correct addition should have been

$$81+162+27+5=275.$$

Furthermore, the testing of the divisibility of

(*Continued on page 192*)

MATHEMATICAL MISCELLANEA

Edited by PHILLIP S. JONES
University of Michigan, Ann Arbor, Michigan

74. A Third Note on the Pythagorean Theorem

Two earlier notes, *Miscellanea 3 and 30* [THE MATHEMATICS TEACHER, XLIII (Oct., 1950), p. 278; and XLIV (Oct., 1951), p. 396], gave proofs, using only those portions of the first book of Euclid which precede the Pythagorean Theorem, of the following proposition and its converse. Here are two additional proofs.

THEOREM: *A triangle whose sides are proportional to 3, 4, 5 is right-angled.*

FIRST PROOF: One can construct a triangle with sides 3, 4, 5 since each of these numbers is smaller than the sum and greater than the difference of the other two. Proving this to be a right triangle is equivalent to showing that if a right angle is included between two sides of length 3 and 4, the hypotenuse is equal to 5.

Consider, therefore, triangle *abc* (Fig. 1) in which *c* is a right angle and the sides $\overline{bc}$ and $\overline{ac}$ are *3* and *4* units respectively. One can, by placing $1+3+5=9$ of these triangles side by side as in Figure 1 form a right triangle ABC in which $\angle C=\angle a$, $\angle B=\angle b$, $\overline{AB}=3\times 3$, $\overline{AC}=3\times 4$.

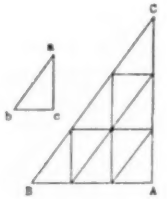


FIG. 1

Moreover, by placing $1+3+5+7=16$ of these triangles as indicated in Figure 2, one can form a right triangle $A_1B_1C_1$ in which $\angle B_1=\angle b$, $\angle C_1=\angle c$, $\overline{A_1B_1}=4\times 3$, and $\overline{A_1C_1}=4\times 4$.

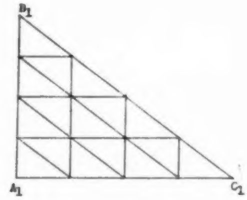


FIG. 2

We can now combine ABC and $A_1B_1C_1$ as in Figure 3 to form BB_1C_1 which has a right angle at B_1 because the angles C and B_1 which are adjacent there are equal respectively to a and b which, in turn, are complementary. Side BC_1 is equal to $9+16=25$ units.

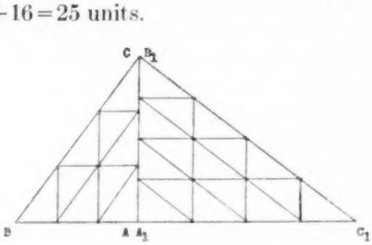


FIG. 3

On the other hand one can juxtapose $1+2+5+7+9=25$ triangles congruent to *abc*, as indicated in Figure 4, to form a triangle DEF in which the sides of the right angle are 5×3 and 5×4, and the hypotenuse $\overline{DF}=5\overline{bc}$.

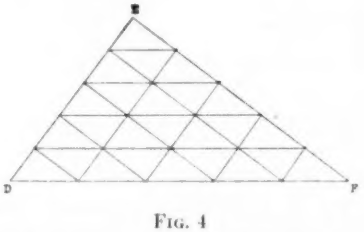


FIG. 4

Let's place the triangle DEF on the triangle BB_1C_1 such that the vertices of the

right angles B_1 and E coincide. $\overline{ED}$ lies along $\overline{B_1B}$ and $\overline{EF}$ along $\overline{B_1C_1}$. DEF is in the position $D'EF'$ as in Figure 5. $\overline{D'F'}$ is parallel to $\overline{BC_1}$ since angles B and D' are both equal to $\angle b$. But triangles BB_1C_1 and $D'EF'$ each contain 25 triangles equal to $\triangle abc$. $\overline{BC_1}$ must then coincide with $\overline{D'F'}$ because if $\overline{D'F'}$, which is parallel to $\overline{BC}$, cut $\overline{EB}$ between E and B, and $\overline{EC}$ between E and C_1, the triangle BB_1C_1 would be larger than the triangle $\overline{DEF}$. Since $\overline{D'F'}$ coincides with $\overline{BC_1}$, $5\overline{bc}=25$, and hence $\overline{bc}=5$.

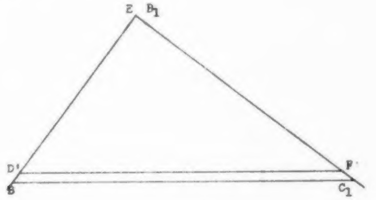


FIG. 5

SECOND PROOF: Let ABC be a triangle in which $\overline{AB}=3a$, $\overline{AC}=4a$, $\overline{BC}=5a$ (Fig. 6). Take $\overline{CE}=\overline{AC}=4a$ and $\overline{BD}=\overline{AB}=3a$.

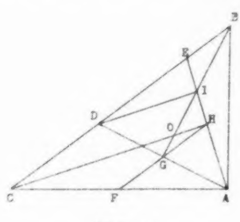


FIG. 6

Mark the points which divide $\overline{CB}$ into 5 parts equal to a. Let D be the second and E the fourth points of division from $\overline{C}$. Draw lines $\overline{AD}$ and $\overline{AE}$. Triangles ACE and ABD are isosceles. Let H, G, and F be the midpoints of $\overline{AE}$, $\overline{AD}$, and $\overline{AC}$. These points then lie on a line parallel to $\overline{BC}$ and we know that $\overline{HG}=\overline{ED/2}=\overline{BE}$. $EBHG$ is therefore a parallelogram. $\overline{EH}$ and $\overline{BG}$ then bisect each other at I and thence $\overline{DI}$ is parallel to $\overline{CH}$ and perpendicular to $\overline{EA}$ since triangle ACE is isosceles.

$\overline{BG}$ is the median of the isosceles triangle ABD. Therefore $\overline{BG}$ is perpendicular to $\overline{AD}$ and hence $\overline{ID}=\overline{IA}$ and triangle DIA is right-angled and isosceles. Thus angle DAE is 45°. Lines BIO and COH are bisectors of the interior angles B and C. They are therefore perpendicular to $\overline{AD}$ and $\overline{AE}$ which meet at a 45° angle and thus the obtuse angle between lines BIO and COH is 135°. The acute angle between them is $(B+C)/2=45°$. Hence $B+C=90°$, and one sees not only that $\angle A=90°$, but also that the point O is the center of the inscribed circle.

VICTOR THÉBAULT
Tennie, (Sarthe)
France

75. A New Solution to an Old Problem

Students of plane geometry can find interesting relationships, applications, and exercises by exploring for themselves the time-honored problem of inscribing a square in a semi-circle.

Some may find the usual solution (Fig. 7) indicated, for example, by Avery. [R. A. Avery, *Plane Geometry* (Rev. ed.). Boston:

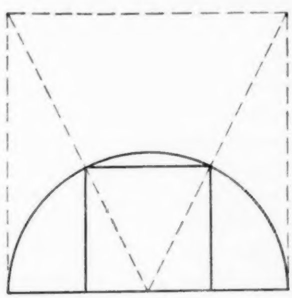


FIG. 7

Allyn and Bacon, 1946, p. 399.] Others may discover intriguing new variations, one of which follows:

Let $x=\frac{1}{2}$ side of square (see Fig. 9), then $2x=$side of square and $r^2=x^2+(2x)^2$, $r^2=5x^2$, $r=x\sqrt{5}$ where r is the radius of the semi-circle. Now draw a semi-circle (Fig. 8) using $3r$ as a new radius. Then h as drawn is the mean proportional between r and $5r$. That is $r/h=h/5r$, or $r=h/\sqrt{5}$, but $r=x\sqrt{5}$ therefore $x\sqrt{5}=h/\sqrt{5}$, $5x=h$, $x=h/5$. Finding $\frac{1}{5}h$ by the

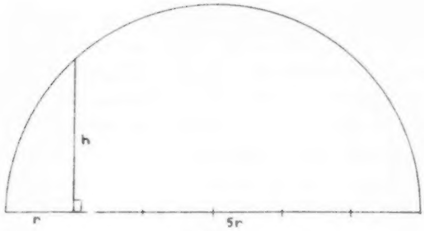


FIG. 8

standard method we can proceed to construct our square as in Figure 9.

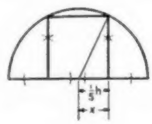


FIG. 9

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76. Continued Fractions and "Rationalizing Numerators."

In *Miscellanea 73* (THE MATHEMATICS TEACHER, vol. XLVI (Feb., 1953), p. 111), Diran Sarafyan used continued fractions derived from a quadratic equation to approximate square roots.

Continued fractions associate interestingly with several other topics in elementary mathematics. This note and the following will try to point out connections with (1) the teaching of radicals and their "standard form," (2) a geometric approach to the irrationality of $\sqrt{2}$ and to successive rational approximations to $\sqrt{2}$, (3) the "golden section," "golden ratio" and the Fibonacci sequence, (4) the solution of quadratic equations.

We frequently teach "rationalizing the denominator" as a trick related to the study of radicals and their "standard" (or "simplest") form without much motivation, application, or even association with such obvious topics as complex numbers and the meaning and importance of irrationals. Rationalizing the numerator instead of the denominator is a useful device which, in its place, provides some motivation, application, and interesting associations. Let's illustrate.

The $\sqrt{2}$ is clearly between 1 and 2. To seek rational approximations to it, isolate the "fractional" part by removing from it the largest integer possible, 1. We then have $\sqrt{2}=1+(\sqrt{2}-1)$, where the quantity in parentheses is less than 1. Focusing our attention upon this part of $\sqrt{2}$, writing it as a fraction and rationalizing its numerator, we have:

$$\begin{aligned} \sqrt{2} &= 1+\frac{(\sqrt{2}-1)}{1} \\ &= 1+\frac{2-1}{\sqrt{2}+1} \\ (1)\qquad &= 1+\frac{1}{\sqrt{2}+1} \end{aligned}$$

Now treating the last denominator in a similar fashion we have:

$$\begin{aligned} \sqrt{2}+1 &= 2+(\sqrt{2}-1) \\ &= 2+\frac{\sqrt{2}-1}{1} \\ &= 2+\frac{2-1}{\sqrt{2}+1} \\ (2)\qquad &= 2+\frac{1}{\sqrt{2}+1} \end{aligned}$$

where the quantity $(\sqrt{2}-1)$ with which we began was less than 1, and our process involved rationalizing the numerator. By substituting (2) in (1) we get

$$\sqrt{2}=1+\cfrac{1}{2+\cfrac{1}{\sqrt{2}+1}}.$$

It should now be clear that if this process were repeatedly applied to the last denominator we would derive the following infinite continued fraction

$$(3)\qquad \sqrt{2}=1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2}+\cdots}}}.$$

If we evaluate this expression at successive steps we obtain the sequence of numbers

$$1+\frac{1}{2}=\frac{3}{2};\quad 1+\cfrac{1}{2+\cfrac{1}{2}}=\frac{7}{5};\quad 1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2}}}=\frac{17}{12}$$

$$1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2}}}}=\frac{41}{29}\,.$$

These fractions are called the *convergents* of the continued fraction and are alternatively above and below the value of $\sqrt{2}$ to which they converge.

A geometric derivation of the above continued fraction expansion for $\sqrt{2}$ was used in one of the earliest geometry books to be published in this country as a proof that $\sqrt{2}$ is irrational (or that the side and diagonal of a square are incommensurable). The book was John Farrar's translation from the French of A. M. Legendre's *Elements of Geometry*.[1] Consider the square on AB and its diagonal AC extended such that $CE=AC$ as shown in Figure 10. A circle drawn with center at C and radius BC will determine point D on AC.

A circular arc with A as the center and radius AD determines F on AB.

The ratio AC/BC is well known to be $\sqrt{2}$. From this diagram and the fact that $CD=CB$ we have

$$\frac{AC}{BC}=\frac{CD+AD}{BC}$$

$$=1+\frac{AD}{BC}$$

$$(4)\qquad =1+\cfrac{1}{\cfrac{BC}{AD}}$$

[1] P. S. Jones, "Early American Geometry," THE MATHEMATICS TEACHER, XXXVII (Jan. 1944), 3, 9.

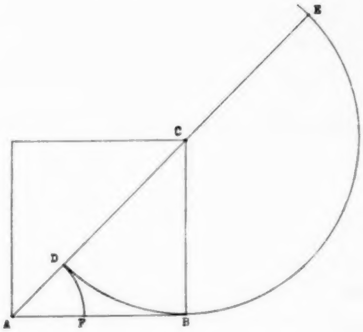

FIG. 10

But AB is tangent to the circle, and AD and AE are segments of a secant, hence $AB^2=AE\cdot AD$, or $AB/AD=AE/AB$. Further, $AE=AD+DE=AD+2BC$, and $BC=AB$. Hence,

$$\frac{AB}{AD}=\frac{BC}{AD}=\frac{AE}{AB}=\frac{AD+2BC}{AB}$$

or

$$\frac{BC}{AD}=2+\frac{AD}{AB}$$

$$=2+\cfrac{1}{\cfrac{AB}{AD}}$$

$$(5)\qquad =2+\cfrac{1}{\cfrac{BC}{AD}}\,.$$

This is a recurrence relation which substituted in formula (4) gives

$$\frac{AC}{BC}=\sqrt{2}=1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2}}}}\cdots,$$

the result we derived previously by "rationalizing the numerator."

The famous "golden ratio," $(\sqrt{5}-1)/2$, is already less than 1 (the term "golden ratio" is also sometimes applied to its reciprocal). To expand it in a continued

fraction one rationalizes the numerator and proceeds as follows:

$$\frac{\sqrt{5}-1}{2}=\frac{5-1}{2(\sqrt{5}+1)}$$
$$=\frac{2}{\sqrt{5}+1}$$
$$=\frac{2}{3+(\sqrt{5}-2)}$$
$$=\frac{2}{3+\frac{5-4}{\sqrt{5}+2}}$$
$$=\frac{2}{3+\frac{1}{4+(\sqrt{5}-2)}}$$
$$\frac{2}{3+\frac{1}{4+\frac{1}{4}+\cdots}}.$$

The successive convergents of this continued fraction are 2/3, 8/13, 34/55, etc. These represent every third term in the sequence of fractions derived from the Fibonacci sequence. This latter sequence is 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, . . . where each new term is formed by adding the two preceding terms. If fractions are formed from each successive pair of numbers in the Fibonacci sequence we have the sequence of fractions referred to; namely,

$$\frac{1}{1}, \frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{8}, \frac{8}{13}, \frac{13}{21}, \frac{21}{34}, \frac{34}{55}, \cdots.$$

Fibonacci first derived his sequence in the second edition (1228) of his *Liber Abaci* in connection with an amusing problem related to the growth of a colony of rabbits.

Not only do continued fractions have a history dating back to early Hindu mathematics and including important contributions to modern analysis where the conditions for and the rapidity of their convergence are studied, but they also have industrial applications in the design of gear trains and the setting up of machines. Perhaps some of our readers will send us further data about pedagogical or industrial uses of continued fractions?

77. Quadratic Equations and Continued Fractions

The two examples of the preceding note illustrate the theorem: *Every quadratic irrationality of the form $(A+\sqrt{B})/C$ where A, B, C are integers and B is not a perfect square can be expanded into a convergent periodic (recurring) continued fraction.*

This, of course, implies that the roots of all quadratics with rational coefficients can be expanded into such continued fractions. One can find procedures for obtaining a continued fraction solution directly from the equation. For example, it can be derived either by the method suggested by Diran Sarafyan in *Miscellanea 73* or by applying the rationalization device suggested in *Miscellanea 76* that a solution of $x^2-ax-b=0$ is given by

$$x=a+\frac{b}{a+\frac{b}{a+\frac{b}{a}+\cdots}}.$$

Mathematical Recreations

(Continued from page 187)

$275_{(16)}$ by 7 should yield

$$18+21+5=44,$$

and $44=6\cdot 7+2$.

The editor of this department hangs his head in shame and offers his thanks to Phineas F. Yoshida of Washington, D. C. and Anthony F. Davidowski of Reading, Pa. for calling attention to his lack of arithmetic skills.

However, paraphrasing Galileo, the editor offers the observation that the method proposed by him for divisibility testing "still works."

A.B.

APPLICATIONS

Edited by SHELDON S. MYERS
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Al. 19 Gr. 10–12 *The Healing or Cicatrization of Wounds*

In the October 1952 issue of this department an appeal was made for information about the use of a formula in predicting the healing of wounds. We were pleasantly surprised to receive letters from seven informed and interested readers. ELIZABETH D. SMITH (Milford High School, Milford, Connecticut), ROBERT V. BELDING (Head of the Mathematics Department, Thomas Carr Howe High School, Indianapolis, Indiana), RALPH E. EKSTROM (Indianapolis, Indiana), and CHARLES D. ARMSTRONG (Parkersburg High School, Parkersburg, West Virginia)—all referred to page 159 in the book *Secondary Mathematics, A Functional Approach for Teachers* by Howard F. Fehr. The letters of Ekstrom and Armstrong also refer to a study reported by Drs. Alexis Carrel, Paul Hartmann and Lecomte du Nouy in Vol. 27, 1918, of the *Journal of Experimental Medicine*. The letter of Armstrong was the most elaborate, consisting of a neatly typed page of description, formulas, and a graph. The formula given was:

$$Q = Pe^{-nr}$$

where P is the original unhealed area measured by a planimeter (on a gauze tracing) in square centimeters, n is the number of days, and r the rate of healing per day. It was pointed out that this formula was derived from the one for continuously compounded interest.

The fifth letter from W. D. REEVE, former editor of THE MATHEMATICS TEACHER and professor emeritus of mathematics education, Columbia University, provided two different sources. One was to a 1916 issue of *Popular Mechanics*, where an article by Dr. Nouy appeared. A more specific reference was made by Dr. Reeve to an article by Dr. Charles N. Moore entitled "Mathematics and the Future" in THE MATHEMATICS TEACHER, April 1929, pp. 203–214. In order to bring present day readers completely up-to-date on this interesting application we will quote the following section from Dr. Moore's article:

> Before the outbreak of the late war, Dr. Alexis Carrel, of the Rockefeller Institute of Medical Research, had noticed in the course of some of his experiments on animals that the rate of healing of a wound seemed to be approximately proportional to its surface area. During the war, as you probably know, he had charge of the large base hospital at Compiègne and therefore had ample opportunity to verify the correctness of his original observation. He soon came to the conclusion that the relationship between the rate of healing of a wound and its surface area could be represented by a mathematical equation. Not having time to work out the details himself, he turned the problem over to one of his colleagues, Dr. P. Lecomte du Nouy, who found after some investigation that the area of the wound surface at any given time could be expressed in terms of the area at the time of the first observation, the interval of time elapsed since that observation, and a certain quantity i known as the index of the wound. This quantity i can in any given case be determined from the first observation and a second observation taken four days later. Once i is determined it is possible to plot the curve of healing and predict the future progress of the wound. It was found by Dr. du Nouy that the curve thus obtained agreed in most instances with the actual course of healing. When there was any marked departure from the theoretical curve it was due to some abnormality in the particular case such as infection in the wound. This made it possible in many cases to detect the presence of infection by comparison with the curve before it was possible to determine this fact by direct examination. Moreover it was found by further investigation that the quantity i mentioned above depends on the age of the patient and the area of the wound surface. Hence for a given age and a wound of a given

size there is a characteristic value of i. A departure from this value reveals the fact that the general condition of the patient is not normal. Further important applications of the curve of healing have been made by other workers, but I have not time to describe them here. Those who are specially interested in the matter can find the material in recent volumes of the *Journal of Experimental Medicine.*

Dr. Moore went on to describe two other mathematical applications—one dealing with economic cycles and the other with the relation between weather and crop yields.

Glenn F. Hewitt of Chicago, Illinois, writes that the formula appears on page 121 of the 17th Yearbook of the National Council of Teachers of Mathematics, *A Source Book of Mathematical Applications:*

$$A = 107 \times 10^{-0.0221t}$$

where A is the area of wound in square centimeters after t days.

Although this reference does not give primary sources, it has the advantages of giving some typical values for the index of the wound and the original unhealed area.

Miss Louisa Alger of Cambridge, Massachusetts writes as follows about the origin of the formula:

Dakin's solution, named for an English-American pharmacist Henry N. Dakin, was developed in World War I in joint work with Dr. Alexis Carrel. The solution was used to irrigate large wound areas and was called then the "Carrel-Dakin Solution." So dependable were the results of using it that a graph could be made predicting the exact date when the final healing would take place. If the actual healing did not follow the graphic prediction, the surgeon knew that either the nurse was not irrigating properly or else there were further complications the surgeon had not diagnosed.

Ar. 27 Gr. 7–12 *Analyzing and Computing the Per Cents of Moisture, Solids, and Fat in Butter*

Your department editor at one time was a food analyst at the Kroger Food Foundation Laboratories in Cincinnati. One of the analyses which was regularly performed was the composition of butter samples in terms of moisture, solids, and fat. The routine of this analysis is so simple that the average junior high pupil can easily understand it. About five grams of a butter sample are accurately weighed in a dry dish which is then dried for two hours in a drying oven. The dish is allowed to cool in a desiccator so as not to pick up moisture and weighed again. The loss in weight is the amount of moisture in the original sample. The fat is then dissolved in ether and the insoluble solids caught in the asbestos filter of a previously-weighed Gooch crucible. The increase in weight of the crucible is the weight of the solids. Since butter contains only moisture, solids, and fat, the percentages of all three can now be computed. Following is a typical set of data for creamery butter, showing the importance, in this case, of skill in using decimals and per cents.

30.538 g. weight of dish+sample
25.673 g. weight of dish
4.865 g. weight of sample

30.538 g. dish+sample
29.944 g. dish+sample after drying
.594 g. weight of moisture in sample

$$\frac{.594}{4.865} \times 100 = 12.2\% \text{ water}$$

35.781 g. crucible+solids after fat is dissolved away
35.523 g. weight of crucible
.258 g. weight of solids

$$\frac{.258}{4.865} \times 100 = 5.3\% \text{ solids}$$

12.2 %	100.0 %
5.3 %	17.5 %
17.5 % moisture+solids	82.5 % fat

Correction: In the January 1953 issue, on page 12, plate 1, of the article "Mathematics in the Skilled Trades" by Carl J. Carlton, change the value of the angle θ on the last line to 8°41′52″. Also a "±" sign should be added in the second fraction of the previous line. This correction was called to our attention by Leo Kessler, High School, Plainville, Connecticut.

WHAT IS GOING ON IN YOUR SCHOOL?

Edited by

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and

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Louisiana State University
Baton Rouge, Louisiana

SPECIAL TOPICS FOR EIGHTH GRADE ARITHMETIC

MOST TEACHERS of effervescent eighth graders in arithmetic are faced perhaps sooner or later with the problem of providing some interesting drill work in the fundamental operational skills. When pupils are tired of the usual textbook practice problems but find word problems difficult because of inadequacy in doing the necessary numerical manipulation, some of the following items can be used to create fun and variety while improving computational skill. Furthermore, an excellent opportunity is afforded for extending the mathematical vocabulary of pupils, and such an opportunity should be used to its full advantage.

The first four topics are usually encountered later in mathematics and certainly cannot be developed very much in grade eight. But if we are to create interest in and appreciation for mathematics, we can well afford, as a by-product of our teaching, to tantalize the inquisitive minds of gifted youngsters. Besides, we can enlighten the slower pupil, in this somewhat terminal period of his study, as to a little of what goes on at higher educational levels.

I. *Symbols of grouping.* Use of parentheses and brackets offer interesting variations on simple problems. Moreover, the pupil will not feel so strange when he meets the idea in algebra. Some possible problems would be:

(a) $7+10-(8+3)=?$

(b) $13-(5+2)+4=?$

(c) $20+1.2+(.7\times 9)=?$

(d) $2\frac{1}{2}+(3\frac{1}{2}-1)=?$

II. *Exponents.* The idea of squaring a number is met in the elementary grades when the areas of squares and circles are studied. Extension of the idea of powers is seldom pursued at that time. To remedy that lack of generalization, to provide another algebraic concept, and ultimately to provide practice in multiplication, problems such as these can be utilized:

(a) $8^3=8\times 8\times 8=?$

(b) $5^4=?$

(c) $11.3^2=?$

(d) $(\frac{1}{2})^5=?$

Some inquisitive soul will no doubt suggest 9^9.

III. *Infinite series.* Our pupils found this exceptionally intriguing, especially when they were told it had applications in radio and electricity if they studied those fields later. Pupils were introduced to series by the classical problem, "How many jumps are necessary for a frog to make to get out of the door of a room, if he jumps, toward the door, half the distance between himself and the door on each jump?"

An immediate purpose for the series problems was practice in addition of fractions. Incidentally, pupils picked up the idea of the existence of infinitesimals and infinity. Some examples of problems are:

(a) $1/2+1/4+1/8+1/16+1/32=?$

(b) $1/2+2/3+3/4+4/5=?$

(c) $1/3+1/9+1/27=?$

(d) $4-2+1-1/2+1/4-1/8+1/16=?$

Extreme caution must be exercised to avoid excessively large denominators.

IV. *Factorials*. This is another of the notions belonging to advanced mathematics in widespread application. But pupils enjoy the multiplication and division (cancellation) afforded. A few of the mathematically mature can follow an example of applying factorials to a simple permutation problem. Some suggestions are:

(a) $4!=1\times2\times3\times4=?$

(b) $10!=?$

(c) $\dfrac{4!\,6!}{3!\,5!}=?$

V. *Statistical data*. For practice in addition, figures gleaned from an almanac are interesting and serviceable. A few suggestions are: finding the total number of postage stamps printed in the United States in the last five years, the cost of television in this country in a specific year, the total number of theaters and amusements by general classes, the number of votes cast for governor in your last state election, and the total population of the five largest cities in the world.

Determining averages based on the various totals gives excellent practice in division. The data also provides opportunity for helping pupils learn to read and round off large numbers.

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A Mathematics Exhibit

Each year for the past several years the students in my classes have been handing in projects as part of their semester's work in mathematics. Each student is allowed to select a project of his choice to work on, provided that this project has some mathematical basis. As a result, the projects turned in are of many types and may include models, charts, and other devices in developing theorems or in application of them; applications of mathematics in the arts, industry, the professions, and advertising, or topics in the history of mathematics, and recreational mathematics.

Two years ago, as we looked over the wealth and beauty of the materials prepared by the students and displayed about the room, it occurred to us that here was a wonderful opportunity to interest others at school in the study of mathematics. We felt that if other students could see examples of the scope and beauty of mathematics shown in this work, they might develop a greater interest in the study of mathematics beyond the minimum requirements for graduation.

The suggestion to the students in my classes that a display of materials be arranged in the school library for all students to enjoy was greeted with much enthusiasm. With the help of the art department, attractive signs were made, projects were grouped and labeled, and the "school public" was invited to visit the exhibit. The interest shown was so great that it was necessary to select guides from each class to take charge of the display and answer the many questions asked by the visitors. Much to our delight, members of the faculty also showed much interest upon visiting the display, and encouraged their students to attend. One of the most frequently heard comments was, "I didn't realize mathematics was so useful."

As a result of the interest shown, we decided that the parents might want to see the work of their children so we arranged to have a display for the Parent-Teachers' Association meeting held the week following the exhibit for the students. Parents were invited to come early in order to visit the exhibit before the meeting. Here again students served as guides and answered questions asked by parents. This undertaking was also very successful. Parents wanted to know more about the mathematics requirements for the different pro-

fessions in which they and their children were interested.

We feel that this exhibit was successful because the students whose work was displayed got much satisfaction from seeing their work admired and enjoyed by others. We have decided to continue this exhibit each spring, not only because it helps create an interest among students and parents in the further study of mathematics, but also because it serves as an incentive for students to hand in neater and more carefully planned work.

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The Students of Senior Arithmetic Class Talk it Over

A Panel Discussion—April 22, 1952

Al: I went into an office one day last fall to apply for a job. The personnel director handed me a paper and said, "Here are a few questions that I want you to answer." I glanced down at the paper, and believe it or not, I felt mighty shaky about getting the job when I faced eight questions in arithmetic. I had had mathematics through trigonometry, but I had not had arithmetic since the eighth grade, and I wasn't sure of my arithmetic. Somewhere in the dim past we had had problems in interest and discount but I felt mighty uncertain about them. That made me realize that I had better take senior arithmetic.

Betty: Do you remember any of the questions asked on the test?

Al: Yes. One was, "If a man received a discount of $18 on a bill of $360, what per cent is this?" I didn't know whether to divide 18 by 360 or 360 by 18. I knew that I had to divide.

Charles: Well, now that you are taking senior arithmetic, is there any question in your mind?

Al: Of course not. It is simply asking, "What part of 360 is 18?" 18/360 can be reduced to 1/20, which is 5%. I remember that when we changed common fractions to decimals we divided, but which to divide by which was not too clear to me.

Betty: Not until this year did I realize that every fraction is an example in division, that is, the numerator is divided by the denominator.

Charles: And it is such a help to reduce the fraction before dividing to get the decimal form.

Dorothy: It seems to me that I have been hearing the word *per cent* ever since I can remember, but I never knew before why 100 is such a convenient denominator to use. It is convenient because our number system is a decimal system; ten is a factor of 100, and so many numbers will divide into it. The per cent sign is simply 100 with the 1 in the middle, one zero in front of it and the other behind it. Since per cent is such an easy fraction to use when there are 100 cents in a dollar, that accounts for its wide use in the business world.

Betty: Yes, and did you know what you were doing when you moved the decimal point from left to right and from right to left? I didn't realize that it was an easy way to multiply and divide by inspection.

Charles: When we were taught to move the decimal point when dividing by a decimal fraction, I did it, but I didn't clearly understand *why* until I took senior arithmetic.

Dorothy: Why do you do it?

Charles: To make the divisor a whole number.

Dorothy: I just wanted to see if you really know. I have found that this course has cleared up so many things for me. They seem so simple if you see the reason behind the work. I am sure that my arithmetic teacher in the grades explained these facts to me, but I must have been too young to take them in.

Charles: That is just the point! I remember that we used to have interest problems in the grades, but not until

this term did I realize that the rate of interest applies to the *interest for one year*. For instance, if you get 6% interest on $300, it is $18 for one year; $9 for 6 months; $4.50 for 3 months, etc.

BETTY: And it is such a help to be able to find 1% of a number by inspection. Then you can find any per cent that you want by simply multiplying; for example: 3 times 1% gives 3%, and $\frac{1}{2}$ of 1% gives $\frac{1}{2}$%. It helps especially in finding fraction of a per cent.

AL: Yes, one question on that test was: "What is $\frac{1}{2}$% of $5,000,000?" I missed it by one zero. Instead of getting $25,000, I got $250,000. Zeros may mean nothing, but add one to the end of a number and see what happens!

BETTY: Can you recall any other questions that were on the test? I am interested; I expect to apply for a job at the same place.

AL: You are? Well, you won't have a bit of trouble after having senior arithmetic. Here is a question that I missed and I am ashamed of it. "If 6 men take 5 days to do a certain job, how long would it take 12 men to do it?"

CHARLES: Don't tell me that you missed that. Did you multiply by two?

DOROTHY: Ha! That is what I might have done. But not now. We have talked so much about examining our answers to see if they are reasonable, that I have formed the habit.

CHARLES: Yes, like the secretary in the insurance office who worked out the premium on a $15,000 policy to be $5,000,000. An insurance man here in Richmond told that story at a public meeting. It really happened.

DOROTHY: I hear that nearly everywhere you go to get a job these days you have to take a quiz in arithmetic.

BETTY: Not only to get a job, but when you join the armed services. What you do on the arithmetic test that you take at the recruiting station decides to a great extent where you will be placed in the Army or Navy. The U. S. Navy offers 34 careers, but you have to be qualified in mathematics to take advantage of them.

CHARLES: I think we have learned much more than just arithmetic in this class. When we studied taxation and filled out Form 1040, the Federal Income Tax Form, we were taught how to fill out forms in general, and learned much about tax laws and regulations.

DOROTHY: Yes, we have received so much information that cannot be found in our textbooks. When the banker and the insurance man came to our class and talked to us, they gave us so much inside information.

BETTY: I liked our visit to the stockbrokers. The stock market reports in the daily papers mean something to me now. And we got such good advice as to investing money and planning for the future.

CHARLES: Our work in insurance was valuable, I think. I never knew before that there were so many kinds of insurance and insurance policies. Our teacher certainly emphasized the importance of reading over policies carefully before signing on the dotted line.

AL: She stressed that fact about any paper that we had to sign.

BETTY: A friend of mine who is studying nursing failed a course this year because she didn't know arithmetic. She said she wishes that she had had arithmetic in her senior year at high school.

AL: Well, all that I can say is that it has certainly been an eye-opener to me. I see a reason for everything that we do now, and I realize the importance of arithmetic in our daily lives. We have truly seen *ARITHMETIC AT WORK*.

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NOTE. For additional references on the numerical solution of equations, Horner's method, Newton's method, etc., see The Mathematics Teacher, March 1951, 44: 204–07.

Conferences and Institutes

Illinois State Normal University announces its **Sixth Annual Spring Conference** on the Teaching of Elementary and Secondary School Mathematics which will be held on **Saturday, April 11.** Principal speakers include Dr. John R. Clark of Teachers College, Columbia University and Mr. Frank Allen of Lyons Township High School at LaGrange, Illinois. A panel at the afternoon meeting will discuss the topic "Evaluating Growth in Mathematics."

The Ohio Council of Teachers of Mathematics will hold a two-day spring conference at **Capital University** on Friday and Saturday, **April 24 and 25** using the theme "Student Discovery of Mathematical Concepts." Principal speakers are Dr. Laura Zirbes of Ohio State University who will speak on "Creative Learning through Discovery" and Dr. C. Louis Thiele of Detroit who will use illustrations from elementary arithmetic in discussing the topic "Student Discovery of Mathematics Concepts." Four discussion groups have been planned for the morning and "Demonstrations of Visual Devices that Aid Student Discovery" will be given at four sectional meetings in the afternoon.

The Western Illinois State College and the public schools of Macomb are sponsoring the **Third Annual Sectional Conference** of the **Illinois Council of Teachers of Mathematics** on Saturday, **May 2.** Main speakers are Dr. Maurice Hartung of the University of Chicago, Dr. Herbert F. Spitzer who is principal of the University Elementary School at Iowa State University, and Professor Daniel Snader of the University of Illinois. The theme of the conference is "How I Teach It."

The **Fourth Annual Conference of the Michigan Council of Teachers of Mathematics** will be held at St. Mary's Lake Camp, near Battle Creek, on **May 1, 2 and 3.** Registration will be held on Friday afternoon, May 1, and the meeting will close with dinner on Sunday, May 3.

The program committee has designated "New Horizons in Mathematics" as the theme of the conference. The guest speaker and consultant will be Dr. John R. Mayor, President of the National Council of Teachers of Mathematics. There will be exhibits, demonstrations, and discussion groups on topics covering a wide range of interests. Among the sessions which have been planned are ones dealing with New Teaching Techniques, Modern Learning Theories and Their Implications for the Teaching of Mathematics, Mathematical Instruments and Field Work, The Use of the Slide Rule, Applications of Mathematics in Industry, The Mathematics Teacher and Guidance, The Teaching of Arithmetic, Mathematical Films, "Dimension X," and Dissecting Polygons.

A review of the growth and accomplishments of this organization might be of interest since this group plans to affiliate at this meeting with the National Council.

At the 1948 Christmas meeting of the NCTM in Columbus, Dr. C. H. Butler of Western Michigan College of Education, Dr. Cleon Richtmeyer of Central Michigan College of Education, and Dr. Phillip Jones of the University of Michigan happened to be discussing some experiments in mathematics education which were being carried on in Michigan and which would be of state-wide interest to many teachers of mathematics if there were an opportunity to exchange information about them. It was observed that there were problems which could be attacked on a state-wide basis and others which were state-wide in occurrence and interest even though requiring solution on the local level. In the course of the discussion the point was made several times that there was no unity or continuity in the planning of the scattered mathematics meetings throughout the state.

In October of 1949, Dr. Raleigh Schorling of the University of Michigan and Miss Irene Sauble, of Wayne University, Detroit, met in Ann Arbor with Drs. Butler, Richtmeyer and Jones to continue this discussion. As an outcome of this meeting, steps were taken to give the teachers of Michigan a means of getting acquainted, exchanging ideas, comparing problems, organizing broader and continuing attacks on common problems, and correlating both the planning and the nature of the programs of the Michigan Education Association Regions, Michigan Schoolmasters Club, The Mathematical Association and other interested groups.

Drs. Schorling and Jones were delegated to organize a group of interested persons who would meet for a week-end conference. Superintendents of schools of all the larger cities of Michigan were invited to send delegates. Also invited were the officers of state mathematics organizations as well as representatives from all the teacher training institutions.

While the organizing group felt they in no way wished to interfere with the program planning, nor dictate the nature of the conference, they did suggest seven questions representing probable areas for consideration and further study, namely—(1) What are the chief problems of mathematics education in Michigan? (2) What are the great needs of mathematics education in Michigan? (3) What experimental and other new projects are now contemplated in the state? (4) Do we have an adequate exchange of information about items (1), (2), and (3) in Michigan? (5) Are there problems which can be attacked on a state-wide basis? (6) Can the planning of mathematics meetings throughout the state be facilitated by an exchange of information and recognition of state-wide problems, and if so, how? (7) Would a State Council

of Mathematics Teachers be helpful, and if so, how?

The first Michigan Conference of Mathematics Teachers was held the week-end of May 6, 7, 8, 1950 at St. Mary's Lake Camp near Battle Creek, Michigan. This conference was attended by 37 teachers of mathematics. The second meeting of the Michigan Conference of Mathematics Teachers was held in May 1951 and included an outside speaker of national reputation who also was a resource person at several smaller group meetings. Attendance at this second meeting increased to around 100. More exhibit material was included as well as opportunity to view recent films and filmstrips. A committee was appointed to draft a constitution with the thought that the adoption of such a constitution would crystallize the conference into a better organized group. Such an organization would have more status than an annual conference of mathematics teachers, and would be in a position to affiliate with the National Council, if the group felt this was desirable. The third conference held in May 1952 saw the adoption of a constitution and recommendations to take steps for affiliation with the National Council.

A record attendance is expected at the fourth annual conference. The camp is beautifully located on St. Mary's Lake and affords excellent recreational facilities. It has accommodations for nearly 150 persons. Meals are served in the main dining hall, and there are dormitory-type sleeping accommodations. The charge is modest. Inquiries and reservations may be sent to Miss Geraldine Dolan, Cass Technical High School, Detroit, Michigan or to Mr. Don Marshall, Dearborn High School, Dearborn, Michigan. This is the best opportunity Michigan affords for its mathematics teachers to get acquainted, exchange ideas, and have a good time together. All mathematics teachers of Michigan and surrounding states, whether in public or private schools, are invited to attend for the full duration of the conference or for as much of it as they can.

The First New Jersey Institute for Teachers of Mathematics will be held this summer under the joint auspices of **Rutgers University**—the State University of New Jersey **and the Association of Mathematics Teachers of New Jersey.** It will meet on the University campus in New Brunswick **during the week of July 12, 1953.**

The plan of the Institute provides opportunity for attendance at two daily study groups, a general lecture and an after-dinner address. In addition, the Department of Mathematics of the University will provide four optional mathematical lectures for those who may be interested. Afternoons will largely be left free for informal discussions, "bull sessions" and for recreation. All of the facilities of the University will be available to participants in the Institute.

Study groups dealing with the following topics are planned: Mathematics in General Education, A One-Year Course in Plane and Solid Geometry, Mathematics for Financial Security, Statistical Concepts, Improving Instruction in Arithmetic, Calculus in the High School, Mathematics for the Gifted Student—Junior High School, Mathematics for the Gifted Student—Senior High School, Modern Concepts in the Teaching of Arithmetic, Mathematics Laboratory: Elementary, Junior and Senior High School Levels. General lectures are planned on the following topics: Teaching the Reading of Mathematics, Mathematics in Insurance, Mathematics in Communication, Mathematics and the Core Curriculum and Articulation of Mathematics in High School and College.

Talks following the dinner meetings will deal with applications of mathematics to industry in the areas of statistical quality control and giant high speed calculators. One evening will be devoted to a discussion of the mathematics tests of the College Entrance Examination Board under the leadership of Dr. Henry S. Dyer, Associate Director of the Board.

Wednesday afternoon, July 15, will be devoted to an excursion. The group will visit the headquarters of the Educational Testing Service in Princeton, and then go on to the seashore for a dip in the ocean, a shore dinner, and such other diversions as individual taste may dictate. The last evening on campus will be marked by a gala closing banquet.

Four members of the Rutgers mathematics department will present a public lecture series during the Institute week on the theme, "Four Great Theorems in Mathematics and Their Significance."

Members of the Institute who desire academic credit may arrange to receive such recognition. Rutgers will grant two points of credit under certain conditions.

A special circular will be issued describing the schedule of the Institute in detail, giving the names of group leaders, speakers and their topics and information as to cost, and manner of enrollment. Persons desiring copies should apply to Dr. Charles H. Stevens, Director of the Summer Session, Rutgers University, New Brunswick, New Jersey. The New Jersey Association cordially invites you to set aside July 13–18, for this Institute and to plan to attend.

The Fifth Annual Institute for Teachers of Mathematics sponsored by the **Association of Teachers of Mathematics in New England** will be held at Colby College at Waterville, Maine, **August 20–27, 1953.** This Institute provides an opportunity for mathematics teachers at all levels to hear outstanding speakers lecture on methods of teaching and laboratory techniques, and to relax and enjoy the recreational facilities of a beautiful and extensive college campus. A variety of teaching aids—commercial, teacher-made and student-made—will be exhibited. Miss Hope Chipman of the University High School, University of Michigan, Ann Arbor, Michigan and Mr. Walter M. Carnahan, Assistant Professor of Education and Mathematics,

Purdue University, Lafayette, Indiana will direct laboratories where mathematical models and devices may be constructed. There will be opportunities for pleasant associations and "shop talk" at tea and during the evening recreation period.

Colby College is located on Mayflower Hill, two miles west of the center of Waterville. With its twenty-one new buildings and six hundred and fifty acres of campus including a six acre artificial pond, the college provides an inspiring and beautiful location for the Institute. Colby College is located near the famed "Belgrade Lakes," and trips to the Lakes as well as to other places of interest have been planned. Waterville is conveniently reached by automobile (on U. S. Route 201), by train (Maine Central Railroad), by plane (Northeast Air Lines), and by bus.

The registration fee for the entire period of the Institute is $10.00. This fee includes admission to all meetings except for the laboratories where an additional $1.50 is necessary to cover the cost of materials. A fee of $1.00 is charged friends and relatives accompanying members of the Institute but not wishing to attend the meetings. The inclusive rate for board, room and operating expenses is $6.00 per day per person.

For further details and a copy of the program, write to Miss Ruth B. Eddy, 666 Angell Street, Providence 6, Rhode Island.

WOULD YOU LIKE TO TEACH ABROAD?

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The following positions are available at the secondary level for teachers over 25 years of age.

	Europe	*Far East*
Mathematics Teacher	9	3
Mathematics and Science Teacher	7	6
Mathematics, Science and Boys' Physical Education	1	–
Mathematics and Manual Arts	–	1

Assignments in Europe may be to schools in France, Germany, Austria, or Trieste; the Far East—Japan, Okinawa and the Philippine Islands. You can select the general area, but the needs of the school system determines the exact location of the job. The foreign differential is paid for Far East duty varying from 10 to 25% of the salary, dependent upon location.

Minimum qualifications are a bachelors degree, 18 semester hours credit in education courses, 15 semester hours in mathematics and/or 10 semester hours each in chemistry and physics, 2 years of public school teaching experience, and possession of a valid state teaching certificate. The maximum age for women is 45 and for men 55.

Families of teachers selected for assignment to Europe are permitted to follow the employee at a later date. The waiting period varies from two months to one year and is entirely dependent upon the availability of family quarters in the area to which the employee is assigned. Teachers in the Far East are not eligible for government family quarters. The employment of a man and wife team is permitted only in Trieste and on Okinawa. Opportunities in both of these areas are very limited.

Qualified candidates are being interviewed at various points throughout the United States from February 20th to April 23rd. Additional information about this program may be obtained from the Overseas Affairs Division, Office of Civilian Personnel, Office, Secretary of the Army, Washington 25, D. C.

AIDS TO TEACHING

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BOOKLETS

B. 127—Elmer's Gyros

B. 128—The Gyroscope through the Ages

Sperry Gyroscope Company, Great Neck, N. Y.

Booklets; Free.

Description of B. 127: This 25-page booklet ($5'' \times 7\frac{1}{2}''$) is one of a series on aeronautical instruments and deals with the directional gyro and the gyro horizon.

Appraisal of B. 127: This is somewhat technical for general use in mathematics classes, but for those few who have a special interest in aeronautics (and sometimes many more pupils than teachers do!) it is a fascinating and well-illustrated booklet.

Description of B. 128: Here we have 36 pages ($4'' \times 8\frac{1}{2}''$) of information concerning various places where gyros occur such as the turning earth, diptera, and spinning tops.

Appraisal of B. 128: There is little mathematics here but a great deal of other science. This booklet could be used as supplementary material to inspire mathematical discussion. There is remarkably little advertising in it.

B. 129—Electronic Analog Computer

Boeing Airplane Company, Seattle 14, Washington

Booklet; $8\frac{1}{2}'' \times 11''$; 12 pages; Free.

Description: This describes the solution of a second-order differential equation, a list of typical applications of the computer, a list of basic and associated equipment and engineering specifications. The back cover shows wiring diagrams for eight typical applications.

Appraisal: Although the actual subject matter is best suited to college classes it still may serve as an inspiration to high school classes to see familiar algebraic expressions in use in the airplane industry. Illustrations of one type of automatic computer are also included.

B. 130—Information Regarding the Operation of the New York Clearing House

New York Clearing House, 77–83 Cedar Street, New York 5, N. Y.

Booklet; $6'' \times 9\frac{1}{2}''$; 8 pages; Free

Description: This describes the origin, operation and functions of the New York Clearing House in serving the banks of that city.

Appraisal: This is a factual pamphlet with no illustrations or attractive titles. It does its job well though and explains in an efficient and clear manner the various functions of a clearing house; this proves to be of more than ordinary interest. This should be used for supplementary reading in more advanced classes in consumer mathematics or business mathematics. There are plenty of large numbers to read on the first couple of pages.

B. 131—Optics and Wheels

Department of Public Relations, General Motors, Detroit 2, Mich.

Booklet; $5\frac{1}{2}'' \times 8\frac{1}{4}''$; 32 pages; Free

Description: This contains a story of ways of producing artificial light, some of the laws of physics which govern light, the

mechanism of the eye, and a discussion of automobile headlights.

Appraisal: This booklet contains excellent illustrations. Those useful to mathematics classes include the law of inverse squares, the equal angles of incidence and reflection, parabolic mirrors and mirages. A good classroom reference book to inspire pupil reports.

B. 132—Journal of Calendar Reform

B. 133—The World Calendar—Stability with Balance

B. 134—Improve the Calendar

The World Calendar Association, Inc. International Building, 630 Fifth Avenue, Suite 903, New York 20, N. Y.

Booklets; Free.

Description of B. 132: This journal seems to contain 48 pages each time ($6'' \times 9''$) and to come out quarterly. It is printed on excellent paper and contains articles advocating the World Calendar.

Description of B. 133: This pamphlet ($5\frac{1}{2}'' \times 8''$, 10 pages) contains a speech describing and attempting to prove the need for the World Calendar.

Description of B. 134: Another pamphlet ($5\frac{1}{2}'' \times 8''$, 12 pages) presenting basic arguments concerning the defects of our present calendar and the advantages of a new one. The center pages contain a comparison of the Gregorian and the World calendars for 1956 which would be useful for bulletin boards.

Appraisals of B. 132 through B. 134: These pamphlets on the World Calendar are excellently written and attractive pieces of printing. Certainly any idea as fundamental in its effects and changes as a new calendar needs a great deal of publicity, argument and money to convince the public that it should be effected. There are good arguments for the change, but hearing or reading them over and over again in so many forms makes one a little tired of them. Nevertheless, this material should certainly be available for reference.

B. 135—The Calendar for Everybody

The World Calendar Association, 630 Fifth Avenue, New York 20, New York.

Book; $5\frac{1}{2}'' \times 8''$; 141 pp., cloth cover; Free.

Description: This book was written by Miss Elisabeth Achelis and was published in 1943 (G. P. Putnam's Sons). It contains considerable material on the history of the compass, clock, and calendar and some account of attempts to introduce a new international calendar through the League of Nations. There is considerable personal history of the author and the thinking she went through before deciding to sponsor a new world calendar and found an association for this purpose.

Appraisal: There is much useful material in this book, but it is diluted by the biographical situation in which the author places. it. On the other hand, this personal style adds to the interest and charm of the writing. The story of a devoted person who follows a single idea with great tenacity illustrates how much such a person can do to further a cause.

FILMS

F. 75—Perspective Drawing

Educational Film Sales Dept., University of California, Los Angeles 24, California.

B&W ($45.00, rental $2.00); 400 feet, 8 min.

Description: The beginning of this film tells how shapes are combined to give the impression of three dimensions in perspective drawings. After showing how the lines of sight to points on an object intersect the picture plane, the discussion deals with one-point, two-point and three-point perspective.

Appraisal: Much of this production is static where it should be dynamic; a series of still pictures could have been done in animation and have been more valuable. For example, the changes from incorrect to correct drawings could have been continuous. There are not many ap-

plications of this film to mathematics classes beyond the use of geometric shapes in practical situations. But then the material most suitable for mathematics films is doubtless the applications not the mathematics itself. A solid geometry class could make a very interesting study of intersecting lines and planes after seeing this film. There is some attempt at humor in the cartoons which is helpful.

F. 76—Geometry in Action

Library Films Inc., 25 West 45th Street, New York 36, New York.

B&W ($37.50); 400 feet, 10 min.

Description: This film was originally made by Bald Eagle Film Company. It shows a railroad with circles and straight lines, autos with curves, and planes and boats. There are trees for symmetry, and apples and pears and honeycombs. Parallel furrows on the farm are followed by machines "built on geometric principles," the cutting of cloth into congruent figures, drafting techniques, and the need for triangles to give stability to figures. Then we have buildings, bridges and microscopes with geometric figures; and finally home uses of geometry including "congruent plates."

Appraisal: The pictures are well chosen and photographed and certainly use ideas within the experience and interest of high school pupils. However, this film, as the description above shows, tries to do too much. By including figures only and little of the real geometry behind the figure, the whole film is shallow and thus uses dishonest motivation. A collection of mathematical words is not mathematics, but gives the appearance of being important. Also, the organization is very poor: there is just one example after another with no clear outline giving the film form and structure. What are the main points it is trying to make? For an introduction the film may be useful if it is carefully buttressed with real teaching of mathematics.

F. 77—Orthographic Projection

McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 36, N. Y.

B&W ($85); 700 ft., 18 min.

Description: After an introduction concerning the use of drawings, this film discusses the geometry of the point, line and plane as seen from different points of view. Then comes a short discussion about the three views used in orthographic projections. A clever demonstration of how one could start with the three views and cut the solid from a solid block is followed by the converse problem of starting with the solid and making the three drawings. In conclusion there is a short review and something more on the use of drawings.

Appraisal: Everything in this film could be done by a model made of three pieces of glass, but not as clearly, simply nor easily. As far as mathematics classes go, this film could be used wherever the relationships between plane and solid geometry are discussed; and this could be anywhere in grades 10 through 12 as far as the approach and maturity of the film are concerned. It is the type of practical application which should certainly be introduced.

F. 78—Geometry

Wholesome Film Service, Inc., 20 Melrose Street, Boston, Mass.

B&W ($30); 800 ft., 30 min.; silent.

Description: A tricky title showing a juggler juggling mathematical figures is followed by short descriptions and still pictures of mathematicians. We then see animation and simple photography illustrating the following ideas: a line in a plane, cube, cylinder, cone, sphere, point, line, curved line, broken line, polygon, lines intersecting in a point, circle, angles, constructions, polygons, triangles, parallel lines, quadrilaterals, angles in circle, tangent to circles, locus, ratio, similar figures,

commensurable and incommensurable quantities, and pratical applications.

Appraisal: This is a very old film and shows it, both photographically and educationally. The photography is quite amateurish and tends to interpret the static diagrams of textbooks quite literally. The definitions, expressions, and symbols used are very much out-of-date and sometimes even incorrect. The order of subjects is illogical, uses too much reading in the captions, and tries to do too much in a single film. This last point could be eliminated by showing short parts of it or breaking it up into small sections. The film is completely useless for a classroom and would not deserve a review except as inspiration for amateur motion picture makers to adapt such material for their own mathematics classes.

F. 79—Helping Children Discover Arithmetic

Audio-Visual Materials Consultation Bureau, College of Education, Wayne University, Detroit 2, Michigan.

B&W ($75, rental $3); 500 feet, 15 min.

Description: This film is directed at the teacher of elementary arithmetic and portrays a classroom situation in the teaching of subtraction. It stresses the inductive method and uses play money, real tickets and medical applicators.

Appraisal: The reactions of the children are good because they are typical. The progress from real objects to abstractions is, of course, admirable and, moreover, shows how this is facilitated by having a useful collection of teaching materials in the corner of the classroom. On the other hand, all the action takes place in a classroom and therefore shows nothing which a good demonstration lesson or class visit could not accomplish. The speed at which the ideas are developed in a slow, meticulous and almost boring fashion may be helpful if the ideas in the film are all new. For groups of teachers away from centers of discussion this film is excellent. To substitute this film for a class demonstration when that is difficult to achieve, is quite acceptable.

F. 80—Maps and Their Uses

Coronet Instructional Films, Coronet Building, Chicago 1, Illinois

B&W ($50) or Color ($100); 400 ft., 10 min.

Description: In presenting the story of a family using maps to select a new home site, this film tells about the following characteristics of maps: the grid on the edge; the symbols and legend of a map; the map scale; the index to places on a map; maps for special uses such as those locating hospitals, railroads, water mains and sewers; contour maps (showing contours on a cone as illustration); and distances and directions on maps.

Appraisal: This is an excellent survey of the most essential concepts needed to interpret and use maps. For mathematics classes the idea of contours is most appropriate. The cone models and the plane representation of lines on the cone could be expanded to aid mathematics teaching. On the other hand, it is perfectly possible for a teacher with a set of road maps for all members of the class to cover much of the same material without a film. A color print would be preferable to a black and white one, since color adds considerable to this particular subject.

F. 81—Your Family Budget

Coronet Instructional Films, Coronet Building, Chicago 1, Illinois.

B&W ($50) or Color ($100); 400 ft., 10 min.

Description: The story in this film begins with a family discussion in which every member wishes to spend the same $20. It is stated that even after a budget is made, planning to spend each amount of money is important. Although a family may keep

a rough budget, a more detailed one is usually useful. After a budget is determined, the responsibility for spending the money and keeping the accounts should be shared by all members of the family.

Appraisal: This whole film is filled with excellent advice, but possibly will not find its greatest usefulness in mathematics classes. Social studies groups may use it more. All films of this type suffer from the same trouble—having the definite amounts of money mentioned now hopelessly out-of-date.

F. 82—Bookkeeping and You

Coronet Instructional Films, Coronet Building, Chicago 1, Illinois.

B&W ($50) or Color ($100); 400 ft., 10 min.

Description: A class of students in bookkeeping think about their various needs for the course and the following uses are brought out: accounts in a grocery store, in farming, in family accounts, in social clubs, in the vocations of bookkeeping and accounting, and in the job of public office holder (to analyze accounts).

Appraisal: Since this is aimed at bookkeeping classes and since only two of the applications will probably appeal to students in mathematics, it is not very useful for these latter groups.

F. 83—What is a Corporation?

Coronet Instructional Films, Coronet Building, Chicago 1, Illinois.

B&W ($50) or Color ($100); 400 ft., 10 min.

Description: Corporations may be used for businesses, towns, schools, hospital, and other groups. It is just one way of organizing a business, but is the one which carries with it the rights of an individual. Other methods of organization are single proprietor and partnership.

Appraisal: As the description above suggests, this is good material for mathematics classes but more especially aimed at business subjects and social studies groups.

F. 84—What Time is It?

Coronet Instructional Films, Coronet Building, Chicago 1, Illinois.

B&W ($50) or Color ($100); 400 ft., 10 min.

Description: A little girl is addressing invitations for her birthday party and begins to notice the need for telling time. Hours and half-hours are discussed, then she goes to bed and has a dream about a clock with removable hands. The analogy with cutting a pie brings us to quarter hours, and uses such expressions as "two quarters past the hour" or "half past." We then follow the time through the day from one evening to the next. The importance of time, the way of telling minutes by counting by fives, the use of a calendar to tell what day it is, and the idea that candles on a cake indicate a number of years are all introduced.

Appraisal: The story of this girl telling time is quite appropriate and tries definitely for audience participation. However, a few questions are raised in the reviewer's mind: Is this subject one which really requires motion and the additional expense of motion pictures? Would not a filmstrip combined with a model be as good, less expensive, and more adaptable in teaching? There is a slight appearance of having padded the material in order to fill up a film to the usual length of 10 minutes. Along with other devices to teach time, this film would be useful, but it is not essential.

F. 85—Per cent in Everyday Life

Coronet Instructional Films, Coronet Building, Chicago 1, Illinois.

B&W ($50) or Color ($100); 400 ft., 10 min.

Description: The first illustration of per cent is that of a club which wishes to buy

(*Continued on page 215*)

DEVICES FOR A MATHEMATICS LABORATORY

Edited by Emil J. Berger
Monroe High School, St. Paul, Minnesota

Anyone who has a learning aid which he would like to share with fellow teachers is invited to send this department a description and drawing for publication. If that seems too time-consuming, simply pack up the device and mail it. We will be glad to originate the necessary drawings and write an appropriate description. All devices submitted will be returned as soon as possible. Send all communications to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

Mechanical Device for Drawing the Sine Curve

The use of graphs in studying the trigonometric functions provides a convenient and understandable means of developing the variation concept as it relates to this class of functions.

When drawing the graph of a trigonometric function it is convenient to find the values of the function for assigned values of the variable angle simply by referring to tables of natural functions and employing the concept of "related acute angle." Then the curve can usually be sketched after plotting enough points to reveal its outline.

A second way of drawing the graph of a trigonometric function is by making use of its line values. This method is completely geometric and does not require the determination of numerical values for plotting points on the curve. The reader is undoubtedly familiar with the procedure. An outline of the steps involved can be found in most standard trigonometry textbooks. The method is of interest here because it is at the foundation of the device for drawing the sine wave described in this article. In fact the device is based on the method. With it the sine curve can be traced out for the interval $0° \leqq \theta \leqq 360°$, where θ is the variable angle. The device can also be used to produce the cosine curve, and there are other uses worth noting, but let us examine its construction first.

The various parts needed to assemble a workable device are diagrammed separately in Figure 1. Every part is shown; parts which are identical are identified with the same letter.

Part (A) is the drawing board; it may be cut from a sheet of $\frac{3}{8}''$ plywood. Its dimensions are $\frac{3}{8}'' \times 10'' \times 24''$. Two $\frac{1}{4}''$ holes 18″ apart must be drilled through it at points centered as indicated.

Part (B) is the driver rod. It may be cut from any type of wood at all, but it must be straight and untwisted; its dimensions are $\frac{3}{4}'' \times 1'' \times 40''$. In the center of the top front edge a rectangular notch $\frac{3}{8}'' \times \frac{1}{4}'' \times 1\frac{1}{8}''$ must be cut. The reason for this will become apparent later in this description.

Parts (C) and (D) are a pair of wheels $4\frac{1}{4}''$ in diameter and $\frac{1}{2}''$ thick. Their edges must be slightly grooved like that of a pulley wheel; a depression 1/16″ deep is sufficient. The grooves can be cut by hand with a small round rat-tail file, or they may be turned with a wood turning lathe. On part (C) central angles of 15° each should be painted as indicated.

Part (E) is the intersection block. It may be made of either aluminum or hardwood; its dimensions are $\frac{1}{2}'' \times 1'' \times 1''$. For best results use aluminum. Three holes must be drilled through the block in such a way that no one of them will intersect either of the other two. Two of the holes must open on the lateral faces. These should be $\frac{1}{8}''$ in diameter and must be perpendicular to the faces on which they open, but they must lie in different planes.

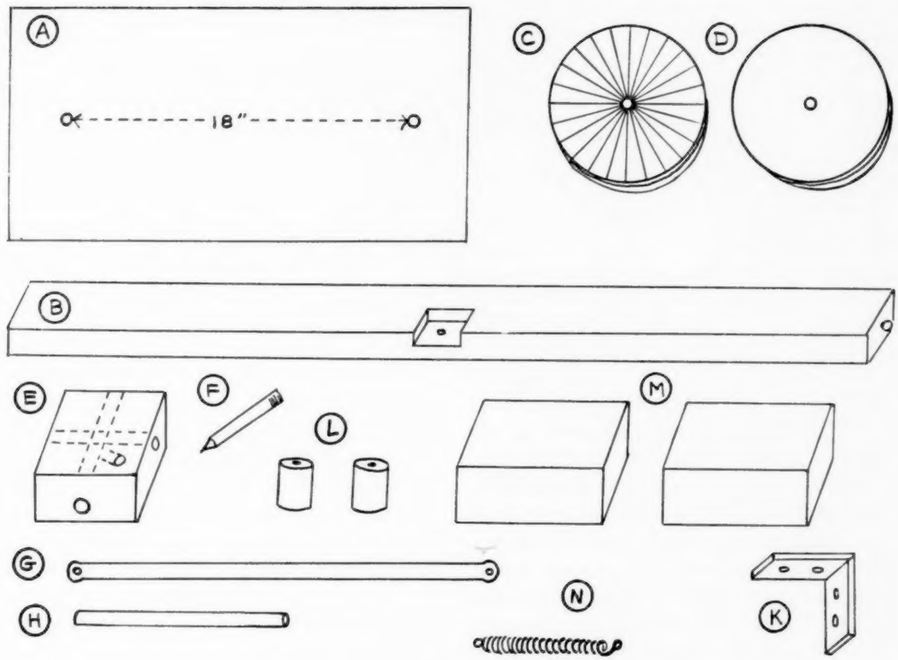

Fig. 1

Let the hole which intersects the two side faces lie above the one that opens on the front and back faces. The diameter of the third hole will depend on the size of the pencil (F) selected. A small bridge talley pencil will work ideally. Drill the hole for the pencil obliquely downward so that when the pencil is inserted the point will be in line with a common perpendicular between the other two holes.

Part (G) is the movable horizontal rod. It may be cut from a piece of welding rod $\frac{1}{8}''$ in diameter; its length should be $18\frac{1}{4}''$. Pass it through the hole which opens on the side faces of the intersection block (E). If it does not slide freely either enlarge the hole or sand the rod. Be careful, however, to keep the tolerance as small as possible. It may only be necessary to use a lubricant in order to obtain the desired free passage. After the rod is in the block, flatten the ends slightly and drill 1/16" holes $\frac{1}{8}''$ from each end. Be sure that these little holes are drilled through the rod in the same direction.

Part (H) is the movable perpendicular rod. It may also be cut from $\frac{1}{8}''$ welding rod; its length should be 7". Attach one end of it to one arm of a piece of angle iron (K) $1\frac{1}{4}''\times 1\frac{1}{4}''$ either by using solder or flattening the end and using small stove bolts. Figure 2 should give the reader the general idea.

Part (L) is a piece of $\frac{3}{8}''$ round dowel $\frac{1}{4}''$ long with a hole drilled along its axis. Two identical pieces are required. Part (M) is a block of wood $\frac{3}{4}''\times 1''\times 2''$. Again, two identical pieces are required. Part (N) is a light steel spring about 2" long.

Materials needed to assemble the device include two $\frac{1}{4}''$ flathead stove bolts $1\frac{1}{2}''$ long, four washers and four nuts to fit these bolts, two screw eyes, two $\frac{5}{8}''$ brads, four 1" wood screws, two $\frac{1}{2}''$ wood screws, and about 6' of soft but fairly heavy string.

Figure 2 gives a fairly accurate repre-

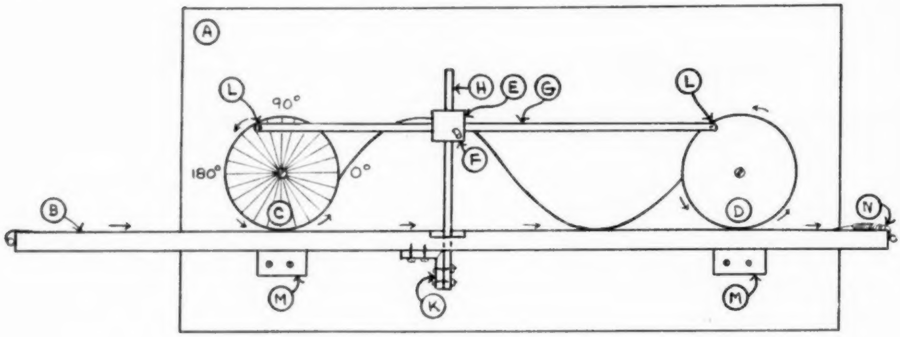

FIG. 2

sentation of how the device is assembled, but a little clarification concerning the order of procedure may be found helpful.

To fasten the wheels (C) and (D) to the drawing board (A) countersink the head of a flathead stove bolt in the center of each wheel, slip a washer over the end of each bolt, pass them through the holes in the drawing board, slip a second washer on each, and then turn two nuts on each. The two nuts should be tightened against each other so that the wheels will always turn free of the drawing board.

Next fasten the blocks (M) to the drawing board with 1″ wood screws in such a way that the driver rod (B) clears between the wheels and the blocks but is not really loose. Note that the driver rod is not fastened to anything. Now fasten the movable horizontal rod (G) to the wheels (C) and (D) with brads using the dowel pieces (L) for clearance washers.

Putting the movable perpendicular rod (H) and the attached angle iron (K) in place requires a little judgment. Begin by locating the intersection block (E) at 0° in such a way that rod (G) is in horizontal position. Adjust the driver rod (B) so that the notch in it is exactly below the intersection block. Then drill a $\frac{1}{8}''$ hole through the bottom of the notched part so that it will line up with the vertical hole in the intersection block. Pass rod (H) through both holes and fasten the angle iron to the driver rod with $\frac{1}{2}''$ wood screws.

Next, fasten screw eyes to the ends of the driver rod (B) and attach the steel spring (N) to the screw eye on the right. Now string the device in the direction of the small arrows. Begin by fastening the string to the screw eye on the left, carry it between the driver rod and wheel (C), wind it once around (C) in a counterclockwise direction and return it between the driver rod and the wheel. Then continue the string to the right along the top of the driver rod and wind it counterclockwise around wheel (D). Finally fasten the string to the end of the steel spring (N). The purpose of the spring is to maintain a constant tension on the string.

To draw the sine curve pull the driver rod (B) to the right and push down slightly on the pencil (F). Because of the lightness of the device this operation usually requires the services of two people —one to run the driver rod and the other to depress the pencil.

The cosine curve can be drawn with the same device if it is restrung with the horizontal rod (G) in the uppermost position possible and the intersection block (E) in such a position that the vertical rod (H) is tangent to (C) on the right. This is also the starting position for drawing the cosine curve for the interval $0° \leq \theta \leq 360°$. This is true because

$$\text{cosine } \theta = \text{sine } (90° + \theta).$$

With the aid of the device it is also possible to illustrate that the direct sine and cosine functions are single valued but that their inverses are not. To demon-

strate the latter fact simply turn the device on end.

THE MATHEMATICS LABORATORY
Monroe High School
St. Paul, Minnesota

THE EIGHT OCTANTS

The device suggested in this note grew out of an attempt by the department editor to help a group of high school juniors visualize the meaning of a first degree equation in three variables. Telling them that this type of equation represents a plane simply wasn't sufficient; they wanted some intuitive verification of the statement. Since this particular group of students was fairly adept with the method of graphing first degree equations in two variables, an approach through graphing seemed both logical and expedient. However, the use of a third reference line and the usual blackboard representation of three mutually perpendicular axes were ideas for which the group did not seem prepared.

After explaining that graphing points in space meant locating them with respect to three mutually perpendicular axes and that three points not in a straight line determine a plane, several students wanted to know whether there was equipment available for graphing a plane similar to the squared blackboard chart used in graphing straight lines. This was the signal for a project assignment. Within a week the class room was literally overrun with gadgets—the students called them "octants." Of the several dozen devices that were produced two came quite close to fulfilling the requirement that had been set for them—that of providing a three dimensional graph "chart" which incorporated the ideas previously learned in connection with the two dimensional blackboard graph chart. In addition, the two devices referred to were completely open; that is, lines (string or knitting needles) could be passed through them in any direction, and all three of the coordinate axes were visible from all possible points of observation.

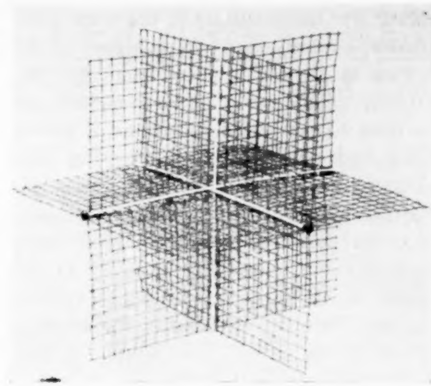

FIG. 3

One of the devices referred to is pictured in Figure 3. It is made of zinc coated steel wire mesh. The openings are $\frac{1}{2}''$ squares. The model illustrated can be produced with seven pieces of this wire mesh cut according to the following dimensions:

(A)	one piece	$12'' \times 12''$
(B)	two pieces	$6'' \times 12''$
(C)	four pieces	$6'' \times 6''$

If the two pieces of set (B) are placed together they will form a 12″ square, and similarly with the four pieces of set (C). The three sets of pieces should be so arranged that each set forms a square which bisects each of the other two squares and is perpendicular to them (Fig. 3). Fasten the various parts in place with thin copper wire. If a really finished job is desired solder the intersections together and remove the copper wire. The completed device is a model which illustrates the way in which three mutually perpendicular planes divide space into eight octants.

Lest the reader get the impression that the white plastic knitting needles which appear in the picture represent the three coordinate axes it should be explained that the needles have been placed in positions which coincide roughly with the axes simply for photographic purposes. They were placed there to help the reader

locate the intersections of the wire mesh planes; actually they are not part of the device at all. Quite apart from this use, however, is the fact that these needles can be used to exhibit the positions of points, lines and planes (two intersecting lines determine a plane) in space. String is also useful. If one wishes to graph a plane—that is, a portion of it—it is only necessary to find three non-collinear points of this plane which lie in any of the coordinate planes. Then the position of the plane in question can be indicated simply by outlining with string the triangle determined by the three points.

Use of the device described above should help high school teachers with the problem of providing a meaningful approach to work with first degree equations in three variables, and incidentally in giving students at this level a glimpse of graphical methods in three dimensions, but the possibilities of the device are by no means limited to these simple cases.

Recently the department editor taught a class in college analytic geometry and this device turned out to be of considerable assistance in developing such concepts as direction cosines, normal to a plane, cylindrical surfaces, etc. The device, however, was not permitted to become a crutch; it was used only in the interest of saving time when a student encountered difficulty in visualizing from two dimensional blackboard drawings.

THE MATHEMATICS LABORATORY
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RATE, BASE, AND PERCENTAGE DEVICE

Attempts at making problems involving rate, base, and percentage understandable for pupils of the junior high school grades have a peculiar way of miscarrying no matter how well they are conceived. Most teachers of arithmetic are undoubtedly acquainted with some satisfactory method of approaching this kind of problems, but there seem always to be pupils in every class who somehow manage to stumble into confusion.

This contributor would like to suggest a device which can be used to assist pupils in visualizing the making of substitutions and selecting the correct operation to perform when the formula method is employed. The device consists of three sections attached by pins so that each section may be rotated independently of the other two (Fig. 4). On one side of each section is a card labeled RATE, BASE, or PERCENTAGE, and on a second side of each section is a card labeled with the arithmetic operation necessary for solution.

Formulas used in setting up and solving problems of this type may be written in the following three ways:

(1) $P \div B = R;$

(2) $P \div R = B;$

(3) $R \times B = P.$

When the device is used the last formula may be introduced to pupils by directing attention to the correspondence between like placed expressions in the following two relations:

(1) $R \times B = P;$

(2) ___% of ___ = ___.

Certainly base does not always follow "of," but on the junior high school level this arrangement is satisfactory as written, and the symbols used in the formula should assist pupils in making correct substitutions. The device will help pupils form a visual image of the processes necessary for solution.

The reader will note that the pupil, can by manipulating this simple device, discover which arithmetic operation must be selected in order to solve a given problem. For example, if the base and percentage are the quantities given for a particular problem and the rate is required, then by rotating the section labeled RATE, the pupil can bring the word DIVIDE into view as well as the directional arrow which shows that the division is to the right, or

FIG. 4

that the base must be divided into the percentage. When solving for any other unknown it is only necessary to rotate the section in question and read the arithmetic operation necessary for solution.

A similar apparatus may be used in physics for problems involving resistance, voltage, and current, where resistance is equal to voltage divided by current; i.e.

$$R = E \div I, \quad \text{or} \tag{1}$$

$$E = R \times I. \tag{2}$$

Materials Needed

2 wooden dowel pins	½"×2½"
3 pieces of pine	1½"×1½"×7½"
6 pieces poster board	1"×7"
woodglue	

Construction Steps

1. Cut three sections of pine 1½"×1½"×7½".
2. Drill ⅝" holes ½" deep into both ends of one section.
3. Drill ½" holes 1" deep into one end of each of the other two sections.
4. Sand and varnish all three sections.
5. Fasten dowel pins into both ends of the center section with wood glue.
6. Label six 1"×7" cards as follows:
   RATE (DESIGNATED BY %)
   BASE (FOLLOWS "OF")
   PERCENTAGE
   DIVIDE→
   DIVIDE→
   MULTIPLY.

WM. F. O'ZEE
University of Texas
Austin, Texas

Aids to Teaching

(*Continued from page 209*)

a radio-phonograph on an installment plan. Then we hear about the way banks use per cent on loans and savings. The mathematical theory of per cent, finally boiling down to the formula $RB = P$, completes the film.

Appraisal: There are some good views of per cent in actual use and some effective arguments for the advantages of summarizing one's ideas in a mathematical formula. This is counterbalanced by poor acting on the part of the student, a mechanical explanation of per cent, a tremendous range of material, the awkwardness of the boy constantly talking to himself, inept expressions like "Is the product of multiplying . . . ," and a real tendency to encourage transposition of letters in formulas. Perhaps mathematics films are trying to do too much and thus are falling short of the mark.

FILMSTRIPS

FS. 161—Orthographic Projection

McGraw-Hill Book Company, Inc., 330 West 42 Street, New York 36, N. Y.

B&W ($4.00); 40 frames.

Description and Appraisal: This filmstrip is planned to accompany the film by the same name (F. 77, in this issue), and is, in fact, organized in the same manner. It is most useful if employed as a follow-up for that film, but can be used separately to good advantage.

BOOK SECTION

Edited by JOSEPH STIPANOWICH
Western Illinois State College, Macomb, Illinois

BOOKS RECEIVED

College

Advanced Mathematics in Physics and Engineering, by Arthur Bronwell, Northwestern University. Cloth, xvi+475 pages, 1953. McGraw-Hill Book Co., 330 West 42nd St., New York 36, N.Y. $6.00.

Mathematics of Finance, by Lloyd L. Smail, Lehigh University. Cloth, x+282 pages, 1953. McGraw-Hill Book Co., 330 West 42nd St., New York 36, N.Y. $4.50.

The Methods of Statistics (Fourth ed.), by L. H. C. Tippett, British Cotton Industry Research Association. Cloth, 395 pages, 1952. John Wiley and Sons, Inc., 440 Fourth Ave., New York 16, N.Y. $6.00.

Introduction to the Foundations of Mathematics, by Raymond L. Wilder, University of Michigan. Cloth, xiv+305 pages, 1952. John Wiley and Sons, Inc., 440 Fourth Ave., New York 16, N. Y. $5.75.

Miscellaneous

Youth and the Community, Part II for Schools. Paper, 64 pages, 1952. Community Chests and Councils of America, Inc., 8 West 40th St., New York 18, N.Y. $0.90 (single copy); $0.85 (50–99 copies); $0.75 (100 or more copies).

REVIEWS

Algebra for Problem Solving (Book One), Julius Freilich, Simon L. Berman, Elsie Parker Johnson. Boston, Houghton Mifflin Co., 1952. 568 pp., $2.88.

A first glance at *Algebra for Problem Solving* might suggest that there is too much material to be given in one year. However, examination shows that the same basic algebra material is made more teachable by using large type, many illustrations and uncrowded pages.

Interest arousing "algebra men" introduce each chapter. Clear concise language on the ninth grade level makes this an outstanding book. Radar, air and space travel problems and interest approaches make an appeal to modern young people.

An innovation is the generous use of red ink. The subtopics in the chapters are not only numbered but each number is set in a red square. Definitions are placed in boxes with two sides outlined with red. There are large red arrows pointing to the things to remember. The examples which stress *What To Do* and *How To Do It* are made to stand out with the use of more red ink. Also the important numbers and expressions are printed in red throughout the book.

Ratio, proportion, variation, square root, radicals, quadratic equations and indirect measurement are discussed more completely than in most ninth grade texts. There is a chapter on reasoning and proof which gives some of the elementary principles of deductive and inductive reasoning.

The authors have provided for individual and group differences in the thousands of exercises and problems, including "Extras for Experts."

The approach and layout of the book represent an innovation in the printing of secondary text books. It needs to be seen to appreciate the effectiveness of using a second color in printing.
—ELIZABETH VAN LIEW, Mooresville, Indiana.

Meaningful Mathematics, H. S. Kaltenborn. New York, Prentice-Hall, Inc., 1951. xiv +397 pages, $4.75.

This book is intended to be a survey course appropriate for meeting the cultural needs of freshman or sophomore college students having no training at all in geometry and very little in high school algebra. The author believes that without further background enough of the subject can be obtained from it to give the student an adequate appreciation of the role of mathematics in civilization. The technical processes of the subject are thus less prominently displayed than is usual, and emphasis is placed more on discussions about mathematics.

The range of subject matter treated can be seen in a listing of the chapter headings, which are as follows: 1) The Nature of Mathematics; 2) The Number System; 3) Functions and Graphical Representation; 4) Algebraic Processes; 5) Consumer Mathematics; 6) Geometric Processes; 7) Numerical Trigonometry; 8) Computation with Logarithms; 9) Introduction to Differential Calculus; 10) The Conic Sections; 11) The Nature of Integral Calculus; 12) Analytic Trigonometry; 13) The Exponential Function; and 14) Further Topics in Algebra and Analytic Geometry.

There is quite a bit of interesting material in this book, especially for students who already have some background in mathematics. The success of this book as a textbook will depend much more upon the teacher who uses it than is the case with traditionally organized books.

There are numerous places where more background knowledge is implied than the fluent mathematics reader might suppose. These deficiencies of background can be overcome only by skillful handling of the textbook on the teacher's part.

The wide freedom usually taken in the reorganization of material for a survey course calls for the utmost care on the part of an author in order not to use mathematical ideas before they are explained. There is always danger that more maturity will be assumed for the student than he really has. There is some indication of this in the Kaltenborn book.

Tests for divisibility of integers by other positive integers are disposed of in one paragraph, while a page and a half of space is devoted to a rather unclear explanation of a game played with matches. Consider the statement "When the number of matches in each row is expressed in the binary scale as shown, the sum is 121." On five of the first 29 pages of the book some use of exponents is made; then on page 30 appears an explanation of the meaning of exponents.

Mention or use of ideas occur frequently, accompanied by the remark that explanation will follow. Such expressions as "we shall see later," seem to appear too frequently. The theorem that the sum of the squares of the cosine and sine of any angle is equal to 1 is used on page 292 and derived on page 321. On page 322 complex numbers are used, but they are defined and discussed on page 366. The law of cosines is obtained (p. 210) by use of a theorem in geometry with which very few high school graduates are familiar, for the theorem is now very seldom included in high school textbooks.

The book is on the whole reasonably well done and carefully edited.

There are between 750 and 800 exercises, many of which call for reports and discussions on topics referred to in other books. Selections for collateral reading are good, but rather limited. The author left out any reference to a number of books that might well have been included.

This reviewer enjoyed the book and recommends it as good reading for any student of mathematics who has completed two or three semesters of advanced high school or elementary college mathematics.—F. W. KOKOMOOR, University of Florida, Gainesville.

Teaching Mathematics in the Secondary School, L. B. Kinney and C. R. Purdy. New York, Rinehart and Company, 1952. xvi+381 pp., $5.00.

This book appears to be well adapted for use as a textbook in a course of prospective teachers of secondary school mathematics. It also contains much information and many suggestions that would be useful to veteran teachers. The early chapters deal with the mathematical needs of modern life, the historical background of the mathematics curriculum, present-day educational problems, and effective ways of directing learning. The mathematics courses of the traditional sequence in high school and the first two years of college are treated in some detail. Then follow four chapters on general mathematics from junior high school through junior college. Included also are chapters on the long-unit assignment, visual aids, tests, and recreational mathematics.

The approach of the book is modern. It contains an excellent treatment of the roles that mathematics should play in the total program of education. The need for a sound mathematics program for future leaders is emphasized, and it has an especially good discussion of mathematics in general education. Outstanding are the many fresh and interesting examples that are given to show how principles of learning can be applied in the classroom. This book should be a welcome addition to the literature in its field.—GILBERT ULMER, University of Kansas, Lawrence.

Trigonometry, Plane and Spherical, with Tables, Lloyd L. Smail. New York, McGraw-Hill Book Company, 1952. xii+406 pp., $3.75.

This is an excellent, extensive text covering all the topics included in the more substantial college trigonometry courses. There is a wealth of material treating both the numerical and the analytical aspects of the subject. Greater attention than usual is devoted to accurate formulation of important definitions. Computation with approximate data is discussed early in the text and, with few exceptions, the author practises what he preaches on this subject. There are many exercises covering a variety of applications including such topics as heights, distances, vectors, aviation, navigation, the terrestrial sphere, great circle sailing, the celestial sphere, and the astronomical triangle. Other features, which do not appear in some elementary trigonometry texts, include a thorough treatment of the solution of the oblique triangle in terms of the right triangle, an excellent analysis of the general sine curve $y=a \sin (bx+c)$, an explanation of the procedure for transforming $a \cos A+b \sin A$ into expressions like $c \cos (A-B)$ or $c \sin (A+C)$, a paragraph on some interesting trigonometric summations, and an excellent chapter on complex numbers. A chapter on logarithms, complete in itself, appears at the end of the text, but may easily be used early in the course for review purposes or as an integral part of the course. A desirable feature is a table early in the text listing the six trigonometric functions to three decimal places for angles at intervals of one degree. The usual four- and five-place tables appearing at the end of the book include a table of squares, square roots, and reciprocals; here again all six functions are listed in the table of natural trigonometric functions. Omitting some or all of the starred sections, the text is adaptable to courses of varying lengths, perhaps three to six semester hours. Starred sec-

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RESEARCH IN MATHEMATICS EDUCATION

Edited by John J. Kinsella
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The Question: What criteria should be used in judging the quality of a program for the preparation and induction of student teachers of secondary school mathematics?

The Study: Rine, T. E. *Criteria for Self-Evaluation of Programs of Student Teaching in Secondary School Mathematics.* Ph.D. dissertation. George Peabody College for Teachers. June 1952.

The purpose of this study was "to develop a specific means of self-evaluation of programs of student teaching in secondary school mathematics." The result was a list of criteria, derived from a study of current literature and the judgment of twenty-four specialists in the teaching of secondary school mathematics. The practicality of the criteria was tested by applying them to the program of a specific institution.

Sixty-three of the seventy-two institutions queried responded with usable replies. Appropriate qualitative and correlational procedures were used to check the validity and reliability of the responses.

Some of the significant findings follow:

1. The courses in mathematics that would be "required of all" before admission to student teaching would be plane trigonometry, analytic geometry, college algebra and differential and integral calculus. The category "Elective—but very highly recommended" would include introduction to mathematical statistics, theory of equations, history of mathematics and solid geometry.

   "Very highly recommended for all" were foundations of arithmetic, algebra, and geometry; fundamental concepts of algebra and geometry; field work in mathematics; college geometry; courses in the teaching of algebra and geometry. A review of secondary school mathematics would be made "elective-recommended for the majority."
2. Of the professional courses only methods of teaching secondary mathematics and principles of secondary education were "required of all." "Very highly recommended for all" were general psychology, educational psychology, educational measurements, organization and function of secondary education, child growth and development, psychology of learning applied to mathematics instruction, philosophy of education and general methods of teaching in the secondary school.
3. Courses in other fields that would be required of all would be English grammar and composition. "Very highly recommended for all" would be physics, speech, English literature and the history of civilization.
4. The professional qualities and abilities that would be required of all previous to admission to student teaching would be fair scholarship in mathematics, ability to organize subject matter of mathematics, fair scholarship in total program of studies, thorough understanding of the objectives of mathematical education and the ability to recognize the place of mathematics in modern civilization.
5. The professional laboratory experiences considered to be "very highly helpful" were supervised observation of secondary school youth in both mathematics classes and other classes and some participation in teaching secondary school youth in mathematics classes.
6. "Very highly helpful" would be the supervisor who
   (a) guides the student teacher in developing a philosophy and objectives for teaching secondary school mathematics
   (b) demonstrates modern teaching practices
   (c) observes student teacher and gives constructive criticism
   (d) gives inspiration and encouragement
   (e) helps student teacher to learn self appraisal
   (f) gradually inducts student teacher into teaching
   (g) helps student teacher develop good discipline and the art of establishing good relations with the students.
7. Practice teaching in both traditional and Track II mathematics was rated most highly among the participation activities of the student teacher.
8. The highest rating among evaluation practices was given to
   (a) informing student teacher early of the evaluation criteria

(b) making evaluation a continuous process
(c) considering the student teacher as a person, teacher, member of a profession and of the community
(d) judging growth of the student teacher in cooperation with the student teacher.

The conclusions and recommendations of the study reflect pretty much the findings listed above. However, the recommendations about the participation period seem to be significant. Additional participation activities recommended are studying the records of individual students; experience in using such instructional aids as projectors, charts, transits, sextants, and slide rules; the construction of various kinds of tests in mathematics; professional reading, attendance at professional conferences and membership in professional organizations; experience in working with mathematics clubs, assemblies and similar activities.

The final recommendation was that a follow-up be made after the student teacher receives a position. This would be of value to the embryo teacher and to those responsible for the improvement of programs of preparing teachers of mathematics.

Book Section

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tions, omitted as a part of a regular course, should provide adequate challenge material for honor students. In addition to its desirable qualities as a text, this book possesses many excellent features as a reference book for students and teachers of secondary and college mathematics.—LAWRENCE A. RINGENBERG, Eastern Illinois State College, Charleston, Illinois.

Graphic Aids in Engineering Computation. Randolph Hoelscher, Joseph Arnold, and Stanley Pierce. New York, McGraw-Hill Book Company, 1952. viii+197 pp., $4.50.

The authors of this textbook are professors of general engineering and engineering drawing. The book is intended for engineering students beyond the freshman year and requires a knowledge of high school algebra, geometry and analytic geometry. An acquaintance with drawing, architects and engineers scales, and engineering problems is assumed. The treatment of slide rules gives procedures for using all scales on a log log rule. The explanation is terse. The fitting of empirical curves is limited to forms $y=mx$, $y=ax^m$, and $y=a(10)^x$, and to experimental methods making the sum of the residuals a minimum. There is no discussion of the limitations of these methods, nor any treatment by moments or least squares. The work on nomograms (alignment charts) gives excellent treatment for parallel, N-shaped, and curved line charts. Further treatment by the use of determinants permits the treatment of 3 variable problems. A chapter is given on graphical methods of constructing a curve $f'(x)$ and $\int f(x)$ for a plotted curve which has no given analytic formula. The book can serve as a handbook and information book; as a textbook it would appear to need supplementary classroom instruction.—HOWARD F. FEHR, Teachers College, Columbia University, New York, New York.

Calculus, Tomlinson Fort. Boston, D. C. Heath and Company, 1951. xii+560 pp., $4.75.

Here is an out-of-the-ordinary presentation of elementary calculus. Definitions are meticulous, statements concerning the validity of formulas precise, and proofs rigorous. The definition of $\lim_{x\to a} f(x)$, for example, is the δ, ϵ definition. The Riemann integral is carefully defined, and there is a thoroughgoing proof that a function continuous over a closed interval is integrable over that interval. The familiar footnote "The proof of this theorem is beyond the scope of this text" is seldom encountered in this book.

There are several striking innovations in approach. The book opens with a chapter on infinite series. Sin x and cos x (also sinh x and cosh x) are first defined by their power series, and the differentiation formulas derived from them (as well as in the usual manner). The logarithmic function is defined (by an integral) without using the exponential function. Length of arc is defined as an integral before differential of arc is considered.

Infinite series, especially power series, are the backbone of the course. If the student can master the first chapter, he is prepared to cope throughout the book with proofs of theorems which might otherwise rest on a foundation of intuition or recourse to a geometric figure. A case in point is a sound proof that for x in radians

$$\lim_{x\to 0} \frac{\sin x}{x} = 1.$$

Two chapters on analytic geometry are included: polar coordinates, and solid analytic geometry. There is a discussion of the Dedekind theory of real numbers in the appendix.

An intelligent student will acquire from this book more than just a technique of manipulation. For the instructor who has been looking for a calculus text with greater-than-ordinary mathematical rigor, this is it.—KATHARINE E. O'BRIEN, Deering High School, Portland, Maine.